ECON 3010 Intermediate Macroeconomic Theory
Solutions to Homework #6

The written questions below should be typed.

1) Chapter 7. Problems and Applications #3.

Solution. To show that the unemployment rate evolves over time to the steady-state rate, let's begin by defining how the number of people unemployed changes over time. The change in the number of unemployed equals the number of people losing jobs (sE) minus the number finding jobs (fU). In equation form, we can express this as:

$$U_{t+1} - U_t = \Delta U_{t+1} = sE_t - fU_t.$$

Recall from the text that $L = E_t + U_t$, where L is the total labor force (we will assume that L is constant). A similar expression can be formed for the change in employment:

$$E_{t+1} - E_t = \Delta E_{t+1} = fU_t - sE_t.$$

See the attached spreadsheet at the end of this document to see that if $s = 0.01$ and $f = 0.19$, then the unemployment rate (U/L) converges to 5% for any initial starting values.

2) Chapter 7. Problems and Applications #5, (a)-(d).

Solution. a) The demand for labor is determined by the amount of labor that a profit-maximizing firm wants to hire at a given real wage. The profit-maximizing condition is that the firm hires labor until the marginal product of labor equals the real wage,

$$MPL = \frac{W}{P}.$$

The marginal product of labor is found by differentiating the production function with respect to labor (see Chapter 3 for more discussion),

$$MPL = \frac{\partial Y}{\partial L} = \frac{2}{3} 5K^{\frac{1}{3}} L^{-\frac{1}{3}}.$$

In order to solve for labor demand, we set the MPL equal to the real wage:

$$\frac{W}{P} = \frac{10}{3} K^{\frac{1}{3}} L^{-\frac{1}{3}}.$$

Solving for L implies that

$$L = \left(\frac{10}{3}\right)^3 K \left(\frac{W}{P}\right)^{-3}.$$

Notice that this expression has the desirable feature that increases in the real wage reduce the demand for labor.

b) We assume that the 27,000 units of capital and the 1,000 units of labor are supplied inelastically (i.e., they will work at any price). In this case we know that all units of each will be used in equilibrium, so we can substitute them into the above labor demand function and solve for $\frac{W}{P}$:

$$\frac{W}{P} = \frac{10}{3} (27{,}000)^{\frac{1}{3}} (1{,}000)^{-\frac{1}{3}} = \frac{10}{3} (27)^{\frac{1}{3}} = 10.$$

In equilibrium, employment will be 1,000, and multiplying this by 10 we find that the workers earn 10,000 units of output. The total output is given by the production function:

$$Y = 5K^{\frac{1}{3}} L^{\frac{2}{3}} = 5(27{,}000)^{\frac{1}{3}} (1{,}000)^{\frac{2}{3}} = 5(30)(100) = 15{,}000.$$

Notice that workers get two-thirds of output, which is consistent with what we know about the Cobb-Douglas production function from chapter 3.

c) The congressionally mandated real wage of 11 units of output is above the equilibrium wage of 10 units of output. Employment is reduced because firms will hire fewer workers at the higher wage. Firms will hire

$$L = \left(\frac{10}{3}\right)^{3} (27{,}000)(11)^{-3} = 751.32$$

workers. The total amount earned by workers is given by the real wage times the number of employed workers: $11 \times 751.32 = 8264.46$. Output is lower and now given by

$$Y = 5K^{\frac{1}{3}}L^{\frac{2}{3}} = 5(27{,}000)^{\frac{1}{3}}(751.32)^{\frac{2}{3}} = 5(30)(82.65) = 12{,}396.8.$$

d) It's not clear if Congress succeeded in helping the working class. The people that are employed receive a higher wage so they are better off. However, the higher mandated wage leads to an increase in unemployment of $1{,}000 - 751.32 = 248.68$ workers. These workers do not receive any wage, so they are clearly worse off.

L	100
f	0.19
s	0.01
E	90
U	10

sE	fU	E	U	U/L
		90	10	0.1
0.9	1.9	91	9	0.09
0.91	1.71	91.8	8.2	0.082
0.918	1.558	92.44	7.56	0.0756
0.9244	1.4364	92.952	7.048	0.07048
0.92952	1.33912	93.3616	6.6384	0.066384
0.933616	1.261296	93.68928	6.31072	0.063107
0.936893	1.199037	93.95142	6.048576	0.060486
0.939514	1.149229	94.16114	5.838861	0.058389
0.941611	1.109384	94.32891	5.671089	0.056711
0.943289	1.077507	94.46313	5.536871	0.055369
0.944631	1.052005	94.5705	5.429497	0.054295
0.945705	1.031604	94.6564	5.343597	0.053436
0.946564	1.015284	94.72512	5.274878	0.052749
0.947251	1.002227	94.7801	5.219902	0.052199
0.947801	0.991781	94.82408	5.175922	0.051759
0.948241	0.983425	94.85926	5.140737	0.051407
0.948593	0.97674	94.88741	5.11259	0.051126
0.948874	0.971392	94.90993	5.090072	0.050901
0.949099	0.967114	94.92794	5.072058	0.050721
0.949279	0.963691	94.94235	5.057646	0.050576
0.949424	0.960953	94.95388	5.046117	0.050461
0.949539	0.958762	94.96311	5.036893	0.050369
0.949631	0.95701	94.97049	5.029515	0.050295
0.949705	0.955608	94.97639	5.023612	0.050236
0.949764	0.954486	94.98111	5.018889	0.050189
0.949811	0.953589	94.98489	5.015112	0.050151
0.949849	0.952871	94.98791	5.012089	0.050121
0.949879	0.952297	94.99033	5.009671	0.050097
0.949903	0.951838	94.99226	5.007737	0.050077
0.949923	0.95147	94.99381	5.00619	0.050062
0.949938	0.951176	94.99505	5.004952	0.05005
0.94995	0.950941	94.99604	5.003961	0.05004
0.94996	0.950753	94.99683	5.003169	0.050032
0.949968	0.950602	94.99746	5.002535	0.050025
0.949975	0.950482	94.99797	5.002028	0.05002
0.94998	0.950385	94.99838	5.001623	0.050016
0.949984	0.950308	94.9987	5.001298	0.050013
0.949987	0.950247	94.99896	5.001038	0.05001
0.94999	0.950197	94.99917	5.000831	0.050008
0.949992	0.950158	94.99934	5.000665	0.050007
0.949993	0.950126	94.99947	5.000532	0.050005
0.949995	0.950101	94.99957	5.000425	0.050004
0.949996	0.950081	94.99966	5.00034	0.050003
0.949997	0.950065	94.99973	5.000272	0.050003
0.949997	0.950052	94.99978	5.000218	0.050002