

ECON 4115/5115

Chapter 8. Exponential Smoothing



Chapter 8. Exponential Smoothing

➤ Simple exponential smoothing

- Exponential smoothing forecasts involve taking a weighted average of past observations
- Weights decline exponentially as observations are more distant
- Simple exponential smoothing (SES) is a compromise between ...
 - the naïve method: $\hat{y}_{T+h|T} = y_T$ &
 - the average method: $\hat{y}_{T+h|T} = \frac{1}{T} \sum_{t=1}^T y_t$
- SES: $\hat{y}_{T+h|T} = \alpha y_T + \alpha(1 - \alpha)^1 y_{T-1} + \alpha(1 - \alpha)^2 y_{T-2} + \dots$
- $0 \leq \alpha \leq 1$ is the smoothing parameter
 - Distant observations get more weight when α is small (i.e., close to zero)
 - Recent observations get more weight when α is close to one
 - $\alpha = 1$ produces naïve forecasts
- SES can be written in weighted-average form: $\hat{y}_{T+1|T} = \alpha y_T + (1 - \alpha) \hat{y}_{T|T-1}$



Chapter 8. Exponential Smoothing

- SES will give flat forecasts: $\hat{y}_{T+h|T} = \hat{y}_{T+1|T}$ for $h = 2, 3, 4, \dots$
 - Flat forecasts are reasonable if there's no trend or seasonality
- You can choose α by minimizing the SSE: $\sum_{t=1}^T e_t^2 = \sum_{t=1}^T (y_t - \hat{y}_{t|t-1})^2$
 - This is a nonlinear optimization problem

➤ Methods with trend

- Holt's component trend method
 - Forecasting equation: $\hat{y}_{t+h|t} = \ell_t + hb_t$
 - Level equation: $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + b_{t-1})$
 - Trend equation: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$
- α and β^* are smoothing parameters for the level and trend, respectively
- Forecasts are no longer flat; increasing in b_t
- Smoothing parameters and initial values, ℓ_0 and b_0 , need to be estimated



Chapter 8. Exponential Smoothing

- Damped trend model may forecast better
 - Adds a damping parameter, ϕ , to Holt's trend method
 - Forecasting equation: $\hat{y}_{t+h|t} = \ell_t + (\phi + \phi^2 + \phi^h)b_t$
 - Level equation: $\ell_t = \alpha y_t + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$
 - Trend equation: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$
- Typically, $0.8 \leq \phi \leq 1$

➤ Methods with seasonality

- Holt and Winters extend Holt's method to handle seasonality
- Seasonality can be quarterly ($m = 4$), monthly ($m = 12$), daily ($m = 7$), etc.
- Forecasting equation and three smoothing equations:
 - Forecasting equation: $\hat{y}_{t+h|t} = \ell_t + hb_t + s_{t+h-m(k+1)}$
 - Level equation: $\ell_t = \alpha(y_t - s_{t-m}) + (1 - \alpha)(\ell_{t-1} + b_{t-1})$
 - Trend equation: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)b_{t-1}$
 - Seasonal equation: $s_t = \gamma(y_t - \ell_{t-1} - b_{t-1}) + (1 - \gamma)s_{t-m}$



Chapter 8. Exponential Smoothing

- The three smoothing parameters $(\alpha, \beta^*, \gamma)$ are usually estimated
- The parameter k is the integer part of $(h - 1)/m$
- There is also a multiplicative version of Holt-Winters model
- Holt-Winters damped trend method with multiplicative seasonality often works well
 - Forecasting equation: $\hat{y}_{t+h|t} = [\ell_t + (\phi + \phi^2 + \phi^h)b_t]s_{t+h-m(k+1)}$
 - Level equation: $\ell_t = \alpha(y_t/s_{t-m}) + (1 - \alpha)(\ell_{t-1} + \phi b_{t-1})$
 - Trend equation: $b_t = \beta^*(\ell_t - \ell_{t-1}) + (1 - \beta^*)\phi b_{t-1}$
 - Seasonal equation: $s_t = \gamma(y_t/(\ell_{t-1} + \phi b_{t-1})) + (1 - \gamma)s_{t-m}$
- Damping parameter is $0 < \phi < 1$

➤ A taxonomy of exponential smoothing methods

- See Table 8.5 with “N”, “A”, “M” and “Ad” notation used in R
- See Table 8.6 for component equations

Chapter 8. Exponential Smoothing

➤ Innovations state-space models for exponential smoothing

- We now express the exponential smoothing model as a state-space model
- Allows us to generate confidence intervals
- $ETS(\bullet, \bullet, \bullet) = ETS(\underline{E}rror, \underline{T}rend, \underline{S}easonal)$ with components:
 - Error = $\{A, M\}$
 - Trend = $\{N, A, A_d\}$
 - Seasonal = $\{N, A, M\}$
- $ETS(A, N, N)$: Simple exponential smoothing with additive errors
 - Component form:
 - Forecasting equation: $\hat{y}_{t+h|t} = \ell_t$
 - Level equation: $\ell_t = \alpha y_t + (1 - \alpha)\ell_{t-1}$
 - State-space form (where $\varepsilon_t \sim NID(0, \sigma^2)$):
 - Measurement or observation equation: $y_t = \ell_{t-1} + \varepsilon_t$
 - State or transition equation: $\ell_t = \ell_{t-1} + \alpha \varepsilon_t$



Chapter 8. Exponential Smoothing

- Smoothing parameter (α):
 - If $\alpha = 0$, the level of the series (ℓ_t) doesn't change over time
 - If $\alpha = 1$, the model collapses to a random walk: $y_t = y_{t-1} + \varepsilon_t$
- See Table 8.7 for ETS additive and multiplicative state-space models

➤ Estimation and model selection

- State-space parameters are typically estimated with maximum likelihood methods
- The AIC, AIC_c, and BIC can be used to select the “best” model
- See R script #11 for an example of ETS estimation, model selection and forecasting

Chapter 8. Exponential Smoothing

➤ Forecasting with ETS models

- Forecasts in R are given by the *forecast()* function
- Point forecasts, $\hat{y}_{T+h|T}$, are given by iterating state-space equations forward
- Prediction intervals are given by $\hat{y}_{T+h|T} \pm c\sigma_h$
 - c depends on the coverage probability
 - σ_h^2 is the forecast variance with formula given in Table 8.8