ECON 4230 Intermediate Econometric Theory
Solutions to Problem Set #3

#1. Gujarati and Porter, 5th Edition, Exercise 8.35.

<u>SOLUTIONS</u>:

a) *Based on the regression of consumption expenditure on real income, real wealth and real interest rate, find out which of the regression coefficients are individually statistically significant at the 5 percent level of significance. Are the signs of the estimated coefficients in accord with economic theory?*

The estimated model is

$$\hat{C}_i = -20.6 + 0.734Y_i + 0.036W_i - 5.52I_i$$
$$(12.8) \quad (0.0138) \quad (0.00248) \quad (2.31)$$

where standard errors are in parentheses. With $n = 54$ and $k = 4$ and a 5% two-tailed test, the critical t value is 2.01. For the slope coefficients, the 5% one-tailed critical value is 1.68. The *t* values for the intercept, *Y*, *W* and *I* are respectively -1.609, 53.376, 14.488, and -2.394. Only the intercept is not significant, and all the signs of the coefficients are in accordance with economic theory.

b) *Based on the results in (a), how would you estimate the income, wealth, and interest rate elasticities? What additional information, if any, do you need to compute the elasticities?*

To calculate the elasticities, we also need the mean of each variable. Then the elasticities can be calculated by

$$Elasticity = \frac{\partial Y}{\partial X}\frac{\overline{X}}{\overline{Y}}$$

$$\epsilon_Y = \frac{\partial C}{\partial Y}\frac{\overline{Y}}{\overline{C}} = 0.734\frac{3215}{2888} = 0.817$$

$$\epsilon_W = \frac{\partial C}{\partial W}\frac{\overline{W}}{\overline{C}} = 0.036\frac{15439}{2888} = 0.192$$

$$\epsilon_I = \frac{\partial C}{\partial I}\frac{\overline{I}}{\overline{C}} = -5.52\frac{1.212}{2888} = -0.00232$$

c) *How would you test the hypothesis that the income and wealth elasticities are the same? Show the necessary calculations.*

We could do a *t* test and see if the two elasticities are significantly different.

$$H_0: \boldsymbol{\beta_2 \bar{Y} - \beta_3 \bar{W} = 0}$$
$$H_A: \boldsymbol{\beta_2 \bar{Y} - \beta_3 \bar{W} \neq 0}$$

The *t* statistic would become

$$\boldsymbol{t} = \frac{\hat{\beta}_2 \bar{Y} - \hat{\beta}_3 \bar{W}}{\sqrt{(\bar{Y})^2 var(\hat{\beta}_2) + (\bar{W})^2 var(\hat{\beta}_3) - 2 \cdot \bar{Y} \cdot \bar{W} \cdot cov(\hat{\beta}_2, \hat{\beta}_3)}}$$
$$= \frac{0.734 \cdot 3215 - 0.036 \cdot 15439}{\sqrt{(3215)^2 \cdot 0.000189 + (15439)^2 \cdot 0.0000062 - 2 \cdot 3215 \cdot 15439 \cdot -0.0000328}} = 22.1.$$

With $n - k = 50$, the 5% two-tailed critical *t* value is 2.01. The *t* statistic exceeds the critical *t* value, so the null hypothesis is rejected in favor of unequal elasticities.

d) *Suppose instead of the linear consumption function estimated in (a), you regress the logarithm of consumption expenditure on the logarithms of income and wealth and the interest rate. Show the regression results. How would you interpret the results?*

The estimated model is

$$\widehat{ln(C)}_i = -0.468 + 0.805 ln(Y)_i + 0.201 ln(W)_i - 0.00269 I_i$$
$$(0.0428) \quad (0.0175) \quad (0.0176) \quad (0.000762)$$

A one percent increase in *Y* causes a 0.805 percent increase in *C*, a one percent increase in *W* causes a 0.201 percent increase in *C*, and a one-percentage point increase in *I* causes a -0.00269 percent change in *C*.

e) *What are the income and wealth elasticities estimated in (d)? How would you interpret the coefficient of the interest rate estimated in (d)?*

The income and wealth elasticities are simply $\hat{\beta}_2$ and $\hat{\beta}_3$ for the log-log model (0.805 and 0.201, respectively). See part (d) for the interpretation of the coefficient on the interest rate.

f) *In the regression in (d) could you have used the logarithm of the interest rate instead of the interest rate? Why or why not?*

No, because you cannot take logarithms of negative numbers.

g) *How would you compare the elasticities estimated in (b) and (d)?*

The elasticities in parts (b) and (d) have the same interpretation, but the ones in part (b) will vary across different values of the explanatory and dependent variables.

h) *Between the regression models estimated in (a) and (d), which would you prefer? Why?*

The double log model makes it easier to interpret and use the elasticities because they are constant over the full range of the data. However, if the best-fitting or theoretical model is linear, then the model in part (a) should be used.

i) *Suppose instead of estimating the model given in (d), you only regress the logarithm of consumption expenditure on the logarithm of income. How would you decide if it is worth adding the logarithm of wealth in the model? And how would you decide if it is worth adding both the logarithm of wealth and interest rate variables in the model? Show the necessary calculations.*

Testing for appropriateness of additional variables can be done with the adjusted R^2 or with *F*-tests. If a variable is added and the adjusted R^2 goes up, then the variable is appropriate. If a variable is added and the adjusted R^2 goes down, the variable is not appropriate.

Using an *F* test, we start by estimating the original model:

$$\hat{C}_i = -0.0731 + 0.996 ln(Y)_i$$
$$(0.0444) \quad (0.00558)$$

To test if the logarithm of wealth is worth adding, calculate the *F* statistic using the goodness of fit with and without the logarithm of wealth:

$$F = \frac{(R^2_{UR} - R^2_R)/1}{(1 - R^2_{UR})/(n-k)} = \frac{(0.99945 - 0.998367)/1}{(1 - 0.99945)/(54-3)} = 100.42$$

With $n - k = 51, m = 1$ and a 5% significance level, the critical *F* value is 4.04 and we reject the null hypothesis that the variable has a zero coefficient. Therefore the logarithm of wealth is worth adding to the model.

Testing if both the logarithm of wealth and the interest rate are worth adding, the *F* statistic is:

$$F = \frac{(R^2_{UR} - R^2_R)/1}{(1 - R^2_{UR})/(n-k)} = \frac{(0.9995597 - 0.998367)/2}{(1 - 0.9995597)/(54-4)} = 67.72$$

With $n - k = 50, m = 2$ and a 5% significance level, the critical *F* value is 3.19. We reject the null hypothesis that the coefficients on the logarithm of wealth and the interest rate are

jointly zero. Therefore, it is worth adding both the logarithm of wealth and the interest rate to the model.

#2. Use the annual data from Problem Set #2 to re-estimate a linear (X-Y) version of Okun's Law over the period 1950-2016. Comment on the results. Then answer the following three questions.

SOLUTIONS:

Table 1. Okun's Law Estimates

Variable	Coefficient Estimates
Intercept	**1.155*** (0.136)**
Real GDP	**-0.363*** (0.034)**
R^2	**0.636**

Notes. Dependent variable is annual U.S. changes in the unemployment rate. Standard errors are in parentheses. * Significant at the 1% level. Sample period 1950 – 2016.**

Real GDP growth explains 63.6% of the variation in the change in the unemployment rate in the U.S. over the sample period 1950-2016. A one percentage point increase in real GDP growth rate leads to a 0.363 percentage point decrease in the unemployment rate.

a) Use an intercept-dummy model to test whether the Financial Crisis (i.e., the Great Recession) led to a significant shift in Okun's Law. Show a scatter plot of your results along with the best-fitting regression lines. Comment on the results.

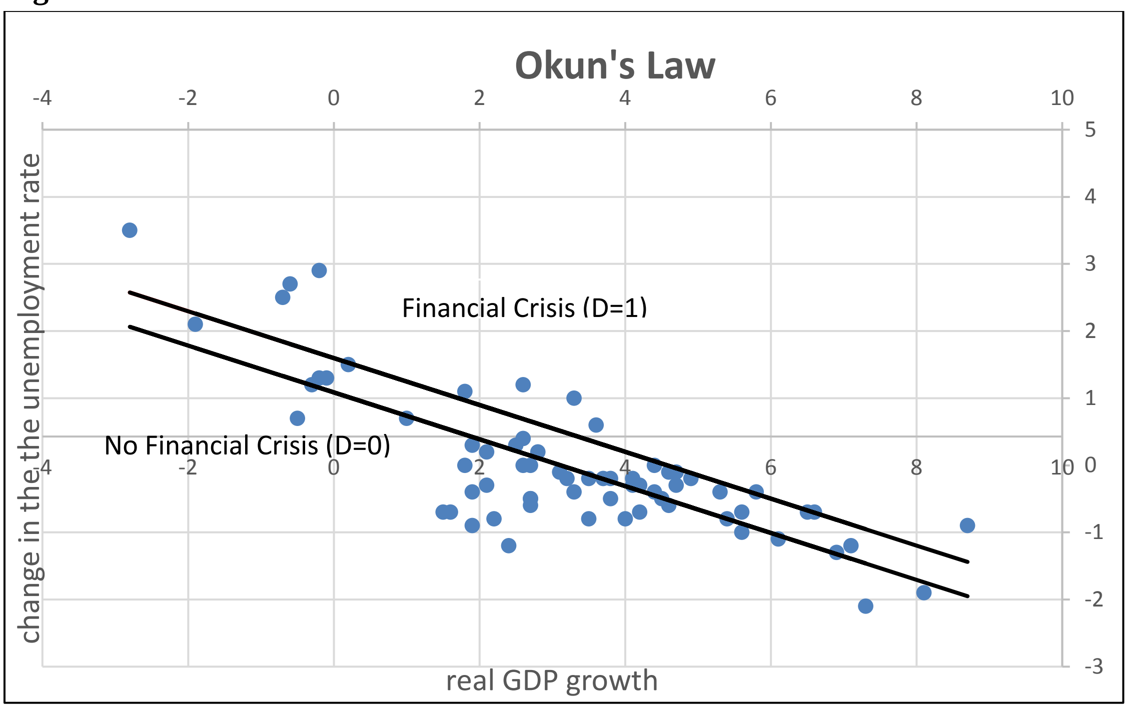
SOLUTION:

Table 2. Okun's Law Estimates with Shift for the Financial Crisis

Variable	Coefficient Estimates
Intercept	**1.086***** **(0.146)**
Real GDP	**-0.349***** **(0.036)**
Financial Crisis Shift	**0.511** **(0.405)**
R^2	**0.645**

Notes. Dependent variable is annual U.S. changes in the unemployment rate. Standard errors are in parentheses. * Significant at the 1% level. Sample period 1950 – 2016. Financial crisis is defined as the years 2008, 2009, and 2010.**

Figure 1. Okun's Law with a Shift for the Financial Crisis



The best-fitting OLS regression line for Okun's Law is higher during the financial crisis (2008-2010), but we fail to reject the hypothesis that the two lines are the same.

b) Use a slope-dummy model to estimate whether there is a structural break after 2000 in Okun's Law. Perform both an unrestricted and restricted general linear F test to for equal slopes across the two periods. Again, show a scatter plot with the two fitted OLS regression lines and comment on the results.

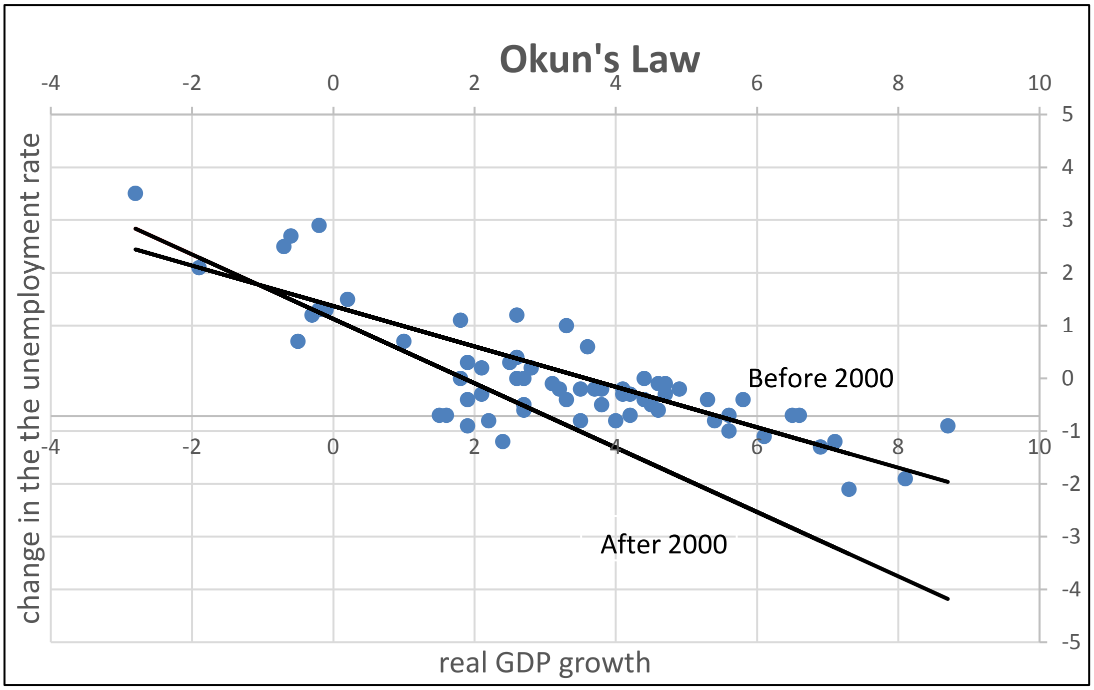
SOLUTION:

Table 2. Okun's Law Estimates with Structural Break for the 21st Century

Variable	Coefficient Estimates
Intercept	**1.370*** (0.150)**
Real GDP	**-0.383*** (0.034)**
21st Century Shift	**-0.243 (0.269)**
21st Century Slope	**-0.227** (0.103)**
R^2	**0.724**

Notes. Dependent variable is annual U.S. changes in the unemployment rate. Standard errors are in parentheses. * Significant at the 1% level. ** Significant at the 5% level. Sample period 1950 – 2016.**

Figure 2. Okun's Law with a Structural Break for the 21st Century



The restricted and unrestricted F tests are designed to give the same results. In both cases, the null hypothesis is H_0: equal slopes and the alternative hypothesis is H_A: unequal slopes. The formula for the restricted version of the F statistic is …

$$\boldsymbol{F} = \frac{(R_{UR}^2 - R_R^2)/m}{(1 - R_{UR}^2)/(n-k)} = \frac{(0.724 - 0.703)/1}{(1 - 0.724)/63} = \mathbf{4.79},$$

where $m = 1$ is the number of restrictions. The unrestricted value for the F statistic is given by the 'test' command in Stata, but it will be equal to the value from the restricted version of the test. At the 5% significance level with $m = 1$ degrees of freedom in the numerator and $n - k = 67 - 4 = 63$ degrees of freedom in the denominator, the critical F value is 4.00. Therefore, we reject the null hypothesis in favor of unequal slopes.

c) Estimate a spline regression model to see if there is a structural break at a critical X value. Choose the best-fitting spline regression across different possible positive integer kinks for real GDP growth. Show the scatter plot with the best fitting regression line and comment on the results. Finally, provide a formal test to see if a spline is necessary.

SOLUTION:

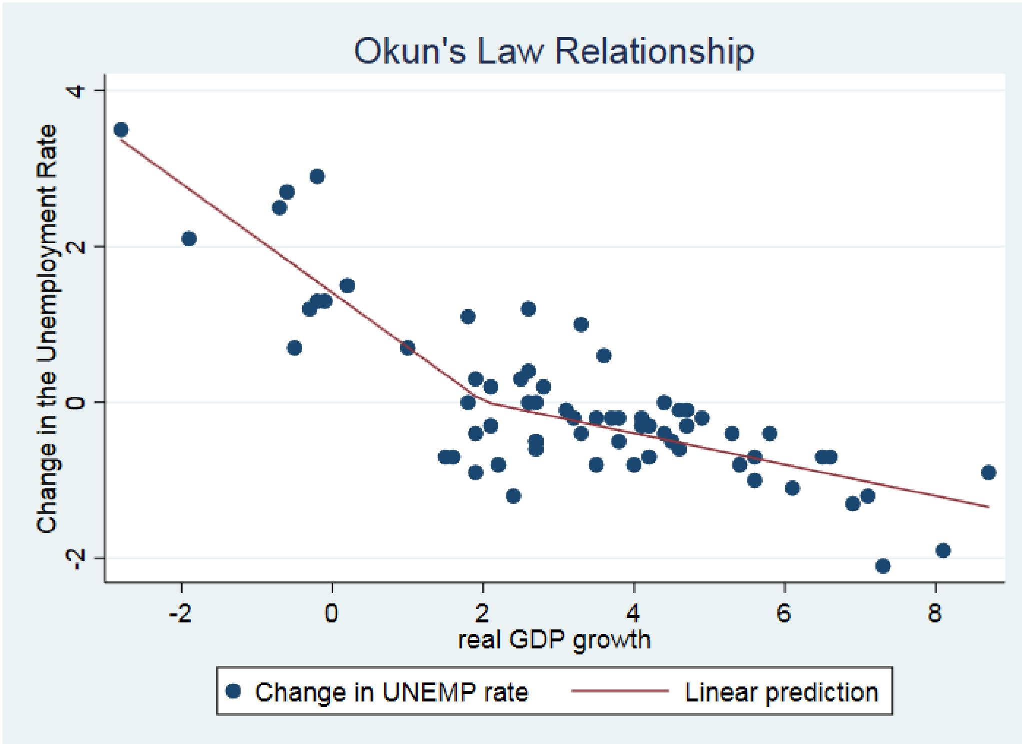
The optimal value for the spline kink that maximizes the R^2 is $X^0 = 2\%$. The scatter plot and best-fitting spline regression results are shown in Table 3 and Figure 3 below.

Table 3. Okun's Law with Spline Regression

Variable	Coefficient Estimates
Intercept	**1.408*** (0.074)**
Real GDP	**-0.700*** (0.074)**
Spline Term	**0.499*** (0.101)**
R^2	**0.737**

Notes. Dependent variable is annual U.S. changes in the unemployment rate. Standard errors are in parentheses. * Significant at the 1% level. Spline knot is located at 2% annual real GDP growth. Sample period 1950 – 2016.**

Figure 3. Okun's Law with a Spline Regression



To see if the spline regression is appropriate, a simple *t* (or *F*) test on the spline term is sufficient. The null hypothesis is H_0: spline coefficient = 0 and the alternative hypothesis is H_A: spline coefficient is not equal to zero. At a 5% significance level, the critical *t* value with $n - k = 67 - 3 = 64$ degrees of freedom is 2.00. The calculated t statistic is

$$t = \frac{\hat{\beta}_3}{se(\hat{\beta}_3)} = \frac{0.499}{0.101} = 4.94.$$

Therefore, we reject the null hypothesis in favor of the spline regression improving the model's fit above and beyond the simple linear X-Y regression.