ECON 4230 Intermediate Econometric Theory

1 Two-Variable Regression: Interval Estimation and Hypothesis Testing

Interval Estimation

- $\hat{\beta}_2$ value is a "point" estimate
- $(\hat{\beta}_2 - \delta, \hat{\beta}_2 + \delta)$ is an "interval" estimate with $1-\alpha$ confidence level
- Significance vs. confidence level
- $\Pr(\hat{\beta}_2 - \delta \leq \beta_2 \leq \hat{\beta}_2 + \delta) = 1 - \alpha$
- Need a probability distribution!

Confidence Intervals

- Standard normal test statistic:
- $Z = (\hat{\beta}_2 - \beta_2)/se(\hat{\beta}_2)$, where $se(\hat{\beta}_2) = \sigma/\sqrt{\sum_i (X_i - \bar{X})^2}$
- ...but, σ is unknown
- Student's t test statistic:
- $t = (\hat{\beta}_2 - \beta_2)/\widehat{se}(\hat{\beta}_2)$, where $\widehat{se}(\hat{\beta}_2) = \widehat{\sigma}/\sqrt{\sum_i (X_i - \bar{X})^2}$
- $\Pr(-t_{\alpha/2} \leq t \leq t_{\alpha/2}) = 1 - \alpha$
- ...and after some light algebra...
- $100(1-\alpha)\%$ confidence level for β_2 is $\hat{\beta}_2 \pm t_{\alpha/2} se(\hat{\beta}_2)$
- Chi-square test statistic:
- $\chi^2 = (n-2)\widehat{\sigma}^2/\sigma^2$
- ...and after some more light algebra, the confidence interval for σ^2 is...
- $\Pr\left[(n-2)\frac{\widehat{\sigma}^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq (n-2)\frac{\widehat{\sigma}^2}{\chi^2_{1-\alpha/2}}\right] = 1 - \alpha$

Hypothesis Testing

- Confidence interval vs. test-of-significance approach
- Steps in the standard approach:
  - Step #1. Form null and alternative hypotheses
  - Step #2. Choose signficance level
  - Step #3. Form test statistic & identify distribution
  - Step #4. Form the decision rule
  - Step #5. Draw conclusion
  - Step #6. Consider possible errors
- p values
- Statistical vs. economic significance
- Analysis of Variance (ANOVA) and the F test
- Prediction
  - $\hat{Y} = \hat{\beta}_1 + \hat{\beta}_2 X_0$
  - Confidence interval: $[\hat{Y} - t_{\alpha/2} se(\hat{Y}), \hat{Y} + t_{\alpha/2} se(\hat{Y})]$
- Reporting regression results