ECON 4230 Intermediate Econometric Theory

1 Multiple Regression Analysis: Estimation

Three-Variable Model: Notation and Assumptions

- $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + u_i$
- Two new classical assumptions:
  - No perfect multicollinearity
  - Model is correctly specified
- Gauss-Markov Theorem: OLS is B.L.U.E.

Partial Regression Coefficients

- Interpretation of β_2 and β_3
- Terminology:
  - "Holding constant..."
  - "Controlling for..."
  - "Accounting for the influence of..."
  - "Filtering out the effect of..."
- Examples:
  - Housing price hedonics
  - Resource curse
  - Empirics of economic growth
- Partial regressions and graphs

OLS Estimation

- The problem & normal equations

- OLS formulae:

$$
\begin{aligned}
\hat{\beta}_1 &= \bar{Y} - \hat{\beta}_2 \bar{X}_2 - \hat{\beta}_3 \bar{X}_3 \\
\hat{\beta}_2 &= \frac{\left(\sum y_i x_{2i}\right)\left(\sum x_{3i}^2\right) - \left(\sum y_i x_{3i}\right)\left(\sum x_{2i} x_{3i}\right)}{\left(\sum x_{2i}^2\right)\left(\sum x_{3i}^2\right) - \left(\sum x_{2i} x_{3i}\right)^2} \\
\hat{\beta}_3 &= \frac{\left(\sum y_i x_{3i}\right)\left(\sum x_{2i}^2\right) - \left(\sum y_i x_{2i}\right)\left(\sum x_{2i} x_{3i}\right)}{\left(\sum x_{2i}^2\right)\left(\sum x_{3i}^2\right) - \left(\sum x_{2i} x_{3i}\right)^2}
\end{aligned}
$$

- Standard errors formulae:

$$
\begin{aligned}
se(\hat{\beta}_2) &= \sqrt{var(\hat{\beta}_2)} = \sqrt{\frac{\sigma^2}{(1 - r_{23}^2) \sum x_{2i}^2}} \\
se(\hat{\beta}_3) &= \sqrt{var(\hat{\beta}_3)} = \sqrt{\frac{\sigma^2}{(1 - r_{23}^2) \sum x_{3i}^2}}
\end{aligned}
$$

Coefficient of Determination, R^2

- Formula:

$$R^2 = \frac{ESS}{TSS} = \frac{\hat{\beta}_2 \left(\sum y_i x_{2i}\right) + \hat{\beta}_3 \left(\sum y_i x_{3i}\right)}{\sum y_i^2}$$

  or

$$R^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \hat{u}_i^2}{\sum y_i^2}.$$

- Adjusted R^2 or $\bar{R}^2$:

$$\bar{R}^2 = 1 - \frac{RSS/(n-k)}{TSS/(n-1)}$$