Econ 5110 Solutions to the Practice Questions for the Midterm Exam

Spring 2012

Real Business Cycle Theory. Consider a simple neoclassical growth model (notation similar to class) where all agents are identical and a representative agent maximizes

$$\sum_{t=0}^{\infty} \beta^t \{\ln(C_t) + \ln(l_t)\}$$

by choosing $\{C_t, N_t\}_{t=0}^{\infty}$ subject to

$$C_t + K_{t+1} \leq s_t K_t^{1-\alpha} N_t^{\alpha}$$

where $l_t = 1 - N_t$, $s_t = s_{t-1}^{\rho} \exp(\epsilon_t)$, $\epsilon_t \sim iid(0, \sigma^2)$ and (s_0, K_0) given. Notice the 100 percent capital depreciation and perfect foresight.

1. (10 pts) Calculate the Euler equation for consumption (C_t) and provide some economic intuition.

   Solution. The Euler equation for consumption is

   $$\frac{1}{C_t} = \beta \frac{1}{C_{t+1}} (1-\alpha) \frac{Y_{t+1}}{K_{t+1}}.$$

   This condition states that at the optimum, the marginal utility of consumption in period t must equal the discounted marginal utility in period $t+1$ from the return on forgone consumption.

2. (10 pts) Calculate the Euler equation for leisure (l_t) and provide some economic intuition.

   Solution. The Euler equation for leisure is

   $$\frac{1}{l_t} = \frac{1}{C_t} \alpha \frac{Y_t}{N_t}.$$

   This condition states that at the optimum, the marginal utility of leisure must equal the marginal utility of consumption derived from the fruits of supplying one more unit of labor.

3. (10 pts) Assume $N_t = 1$ for #3 - #7. Solve for the steady-state values of K and C.

   Solution. The steady-state system is

   $$\begin{aligned} 1 &= \beta(1-\alpha) K^{-\alpha} \\ C &= K(K^{-\alpha} - 1). \end{aligned}$$

   Solving for K and C gives

   $$\begin{aligned} K &= [\beta(1-\alpha)]^{1/\alpha} \\ C &= K[\beta^{-1}(1-\alpha)^{-1} - 1]. \end{aligned}$$

4. (10 pts) Linearize the system.

<u>Solution</u>. The linearized system is

$$\begin{aligned}
\hat{C}_t &= -\frac{K}{C}\hat{K}_{t+1} + \frac{Y}{C}\hat{s}_t + (1-\alpha)\frac{Y}{C}\hat{K}_t \\
\hat{C}_t &= \hat{C}_{t+1} + \alpha\hat{K}_{t+1} - \hat{s}_{t+1} \\
\hat{s}_{t+1} &= \rho\hat{s}_t + \epsilon_{t+1}.
\end{aligned}$$

5. (10 pts) Solve for the reduced-form solution for the state variables.

<u>Solution</u>. The VAR solution for the state variables is

$$\begin{aligned}
\hat{K}_{t+1} &= \left[(1-\alpha) + K^{-\alpha}\right]\hat{K}_t - \left[(1-\alpha)K^{-\alpha}\right]\hat{K}_{t-1} + [1 - 2K^{-\alpha}]\hat{s}_t - [K^{-\alpha}]\hat{s}_{t-1} \\
\hat{s}_{t+1} &= \rho\hat{s}_t + \epsilon_{t+1}
\end{aligned}$$

6. (10 pts) Solve for the reduced-form solution for C_t and $Y_t (= s_t K_t^{1-\alpha} N_t^{\alpha})$ as function of the state variables.

<u>Solution</u>. Using the linearized equations, we have

$$\begin{aligned}
\hat{C}_t &= -[\alpha + \frac{K}{C}]\hat{K}_t + [(1-\alpha)\frac{Y}{C}]\hat{K}_{t-1} + \hat{s}_t + [\frac{Y}{C}]\hat{s}_{t-1} \\
\hat{Y}_t &= (1-\alpha)\hat{K}_t + \hat{s}_t.
\end{aligned}$$

7. (10 pts) Contrast the economic incentives and responses of the representative agent when she faces a positive transitory ($\rho = 0$) technology shock versus a permanent ($\rho = 1$) one.

<u>Solution</u>. When the technology shock is completely temporary ($\rho = 0$) and catches the agent by surprise the agent will work harder in the current period (and later return to the normal work effort) to take advantage of the higher return to labor and smooth the resulting gains over her lifetime by saving (investing) the extra output for later consumption. How much extra effort the agent is willing to supply in the current period depends on her intertemporal elasticity of labor (i.e., willingness to substitute labor across time periods in response to changes in relative returns to working). When the technology shock is permanent ($\rho = 1$), the returns to labor and capital both permanently rise. The intertemporal incentive to work harder today is now dampened because the return to labor will be high in future periods as well. The agent will work harder in the current period and invest some of the proceeds, which will facilitate a permanently higher standard of living (consumption). However, work effort will eventually return to the pre-shock level as the income effect begins to cancel out the incentive to intertemporally substitute labor for leisure. The capital stock, consumption and output will all be permanently higher in the long run, while labor hours will return to their previous steady-state level.

8. (10 pts) Assume that all variation in hours worked happens at the extensive margin. Find the Euler equation

for leisure (or hours worked) for the representative agent and discuss how it will tend to amplify hours worked responses to technology shocks.

Solution. The Euler equation for leisure is

$$1 = \frac{1}{C_t} \alpha \frac{Y_t}{N_t}.$$

Using Hansen's (1985) model, the utility function for the representative agent will be linear in leisure. As a result, the intertemporal elasticity of substitution for labor will be infinite. A positive technology shocks that raises the return to working will therefore cause a large response in the supply of labor.

9. (10 pts) Now assume that government spending is exogenous and is a pure resrouce drain (i.e., does not have any positive imapct on agents' well being). Find the Euler equation for hours worked and discuss how this may act to resolve the Dunlop-Tarshis puzzle in the standard RBC model.

   Solution. The Euler equation for leisure is

   $$\frac{1}{l_t} = \frac{1}{C_t^p} \alpha \frac{Y_t}{N_t}.$$

   where C_t^p is private consumption. A positive technology shock will lead to outward shifts in the labor demand curve but positive government spending shocks will lead to outward shifts in the labor supply curve. With an appropriate mix of both shocks, the model will predict a weak correlation between real wages and hours worked (i.e., Dunlop-Tarshis observation).

10. (10 pts) Now assume a two-country extension of the simple RBC model above where capital is perfectly mobile and technology shocks are contemporaneously uncorrelated but gradually spillover to the other country. Discuss (in words) how a positive technology shock in the foreign country would impact C_t, Y_t and K_t in the domestic country.

    Solution. When the technology shock hits the foreign country, savings (investment) would initially flow from the domestic country to the foreign country. This would reduce the domestic capital and domestic output but the return on foreign investment would allow for higher consumption that would be smoothed over time. As the technology shock gradually spilled over to the domestic country, domestic investment and domestic output would begin to rise again. If the technology shock were transitory but persistent, C_t, Y_t and K_t would gradually return to their previous steady-state levels as the initial shock died out.

Modified Cobweb Model. Let the demand for a good (d) be given by

$$d_t = \alpha_0 - \alpha_1 p_t + \alpha_2 y_t + \alpha_3 E_t y_{t+1} + \epsilon_t^d; \qquad \text{(Demand)}$$

supply (s) be given by

$$s_t = \beta_0 + \beta_1 E_{t-2} p_t + \epsilon_t^s, \qquad \text{(Supply)}$$

where p is price; income (y) is exogenous and follows a mean-zero first-order autoregressive process with persistence parameter λ_y; E_{t-j} is the expectations operator conditional on time $t-j$ information; ϵ_t^d and ϵ_t^s are mean-zero, mutually independent white-noise error terms; all α's and β's are positive; and the market clears (i.e., $d_t = s_t$, for all t).

1. (10 pts) Provide an economic interpretation for the two modifications to the basic cobweb model.

   (a) Solution. The first modification is the addition of y_t and $E_t y_{t+1}$ to the demand function. The economic interpretation is that current and expected future income increase permanent income and the demand for the good. The second modification is that supply depends on the two-period (as opposed to one-period) ahead forecast of price. The economic interpretation is that the good has a two-period gestation period before it is available for the market.

2. (10 pts) Find the steady state.

   (a) Solution. The steady state is given by

$$p = \frac{\alpha_0 - \beta_0}{\alpha_1 + \beta_1} \text{ and } d = s = \frac{\beta_0 \alpha_1 + \beta_1 \alpha_0}{\alpha_1 + \beta_1}.$$

3. (10 pts) Find the equilibrium under naïve expectations. Describe the transition dynamics in words and using a diagram.

   (a) Solution. The equilibrium under naïve expectations is

$$p_t = \alpha_1^{-1}[(\alpha_0 - \beta_0) + (\alpha_2 + \alpha_3) y_t - \beta_1 p_{t-2} + (\epsilon_t^d - \epsilon_t^s)],$$

   for $t = 2, ..., T$ given p_0. Assuming $\alpha_1 > \beta_1$, the equilibrium involves a two-period price cycle that converges to the steady state, p.

4. (10 pts) Find the fundamental rational expectations equilibrium (REE).

   (a) Solution. The REE is

$$p_t = p + \delta y_{t-2} + \eta_t,$$

where

$$\begin{aligned}\delta &= \frac{(\alpha_2+\alpha_3\lambda_y)\lambda_y^2}{(\alpha_1+\beta_1)} \text{ and} \\ \eta_t &= \alpha_1^{-1}[(\epsilon_t^d-\epsilon_t^s)+(\alpha_2+\alpha_3\lambda_y)(\lambda_y\epsilon_{t-1}^y+\epsilon_t^y)].\end{aligned}$$

5. (10 pts) Is a rational bubble possible in equilibrium?

   (a) Solution. A rational bubble equilibrium must satisfy

$$(p_t+B_t)=\alpha_1^{-1}[(\alpha_0-\beta_0)+(\alpha_2+\alpha_3\lambda_y)y_t-\beta_1E_{t-2}(p_t+B_t)+(\epsilon_t^d-\epsilon_t^s)],$$

   which implies that

$$B_t=-\frac{\beta_1}{\alpha_1}E_{t-2}B_t.$$

   Taking expectations conditional on time $t-2$ information implies that

$$E_{t-2}B_t=-\frac{\beta_1}{\alpha_1}E_{t-2}B_t \Longrightarrow B_t=E_{t-2}B_t=0.$$

   Therefore a rational bubble is not possible.

6. (10 pts) Under what conditions is the REE stable under least squares learning?

   (a) Solution. The perceived law of motion (PLM) is

$$p_t=\pi_0+\pi_1y_{t-2}+v_t.$$

   The actual law of motion (ALM) is

$$\begin{aligned}p_t &= \alpha_1^{-1}[(\alpha_0-\beta_0)+(\alpha_2+\alpha_3\lambda_y)y_t-\beta_1E_{t-2}p_t+(\epsilon_t^d-\epsilon_t^s)] \\ &= \alpha_1^{-1}[(\alpha_0-\beta_0)+(\alpha_2+\alpha_3\lambda_y)y_t-\beta_1(\pi_0+\pi_1y_{t-2})+(\epsilon_t^d-\epsilon_t^s)] \\ &= \alpha_1^{-1}[(\alpha_0-\beta_0)+(\alpha_2+\alpha_3\lambda_y)\lambda_y^2y_{t-2}-\beta_1(\pi_0+\pi_1y_{t-2})]+\eta_t \\ &= \tilde{\pi}_0+\tilde{\pi}_1y_{t-2}+\eta_t.\end{aligned}$$

   The REE is stable under learning if the following differential equations are locally, asymptotically stable:

$$\begin{aligned}\frac{d\pi_0}{d\tau} &= \tilde{\pi}_0-\pi_0=\alpha_1^{-1}[(\alpha_0-\beta_0)-\beta_1\pi_0]-\pi_0 \text{ and} \\ \frac{d\pi_1}{d\tau} &= \tilde{\pi}_1-\pi_1=\alpha_1^{-1}[(\alpha_2+\alpha_3\lambda_y)\lambda_y^2-\beta_1\pi_1]-\pi_1\end{aligned}$$

   This condition is satisfied if $-(1+\beta_1/\alpha_1)<0$. The REE is stable under learning because $\beta_1>0$ and $\alpha_1>0$.