Econ 5110 Solutions to the Final Exam

Spring 2009

1. Staggered Prices and Macro Dynamics. (50 pts) Consider the following price setting equation

$$x_t = bx_{t-1} + (1-b)E_{t-1}x_{t+1} + \gamma[bE_{t-1}y_t + (1-b)E_{t-1}y_{t+1}] + \epsilon_t$$

where x_t is the price, y_t is the output gap, ϵ_t is mean-zero price shock, t indexes six-month periods, $\gamma > 0$, and $0 \leq b \leq 1$. Prices are staggered – half the firms in the economy set year-long prices at time t while the other half set prices at time $t+1$. The overall price level is an average of all prices at any period t:

$$p_t = 0.5(x_t + x_{t-1}).$$

Aggregate demand is given by a linearized version of the quantity equation

$$y_t = m_t - p_t + v_t$$

where v_t is a mean-zero demand shock. The monetary authorities set the money supply, m_t, to accomodate changes in the price level:

$$m_t = gp_t,$$

where $0 \leq g \leq 1$.

(a) (10 pts) Find the (reduced-form) rational expectations equilibrium for x_t when $b = 1$.

Solution. When $b = 1$, the system can be reduced to

$$\begin{aligned} x_t &= x_{t-1} + \gamma E_{t-1}y_t + \epsilon_t \\ &= x_{t-1} + \gamma E_{t-1}[(g-1)0.5(x_t + x_{t-1}) + v_t] + \epsilon_t \\ &= x_{t-1} + \gamma[(g-1)0.5(E_{t-1}x_t + x_{t-1})] + \epsilon_t. \end{aligned}$$

Taking expectations conditional on time $t-1$ information gives

$$E_{t-1}x_t = x_{t-1} + \gamma[(g-1)0.5(E_{t-1}x_t + x_{t-1})]$$

or

$$E_{t-1}x_t = \left[\frac{1+\gamma(g-1)0.5}{1-\gamma(g-1)0.5}\right] x_{t-1} = cx_{t-1}.$$

Substituting back into the structural equation for x_t gives

$$\begin{aligned} x_t &= x_{t-1} + \gamma[(g-1)0.5(cx_{t-1} + x_{t-1})] + \epsilon_t \\ &= [1 + \gamma(g-1)0.5(1+c)]x_{t-1} + \epsilon_t \\ &= cx_{t-1} + \epsilon_t. \end{aligned}$$

(b) (10 pts) Find the (reduced-form) rational expectations equilibrium for x_t when $b = 0$. Use undetermined coefficients and a guessed solution of the form $x_t = ax_{t-1} + \epsilon_t$.

Solution. When $b = 0$, the system can be reduced to

$$\begin{aligned} x_t &= E_{t-1}x_{t+1} + \gamma E_{t-1}y_{t+1} + \epsilon_t \\ &= E_{t-1}x_{t+1} + \gamma E_{t-1}[(g-1)0.5(x_{t+1} + x_t) + v_{t+1}] + \epsilon_t \\ &= E_{t-1}x_{t+1} + \gamma[(g-1)0.5(E_{t-1}x_{t+1} + E_{t-1}x_t)] + \epsilon_t. \end{aligned}$$

Substituting in the guess solution gives

$$x_t = a^2 x_{t-1} + \gamma[(g-1)0.5(a^2 x_{t-1} + ax_{t-1}) + \epsilon_t,$$

which after re-arranging gives

$$x_t = \left[a^2 + \gamma(g-1)0.5a(1+a)\right] x_{t-1} + \epsilon_t.$$

Matching up coefficients, produces the following quadratic equation in a

$$(1+d)a^2 + (d-1)a = 0,$$

where $d = \gamma(g-1)0.5$. Solving for a produces two roots:

$$a = 0 \text{ or } a = \frac{1-d}{1+d}.$$

The second root can be ruled out because $d < 0$, which implies that $abs(a) > 1$. Therefore the RE equilibrium solution is $x_t = \epsilon_t$.

(c) (10 pts) In general, how do price dynamics differ when $b = 0$ versus when $b = 1$?

Solution. Price dynamics are more volatile and persistent when $b = 1$ and firms look back when setting prices. The AR(1) process $x_t = ax_{t-1} + \epsilon_t$ will have greater variance and persistence than the original shock. When $b = 0$ and firms only look forward, $x_t = \epsilon_t$ and price volatility is equal to the exogenous shocks (i.e., shocks are not propagated through the economy).

(d) (10 pts) How do the equilibrium dynamics differ under naive expectations?

Solution. Under naive expectations, we have

$$\begin{aligned} x_t &= bx_{t-1} + (1-b)x_{t-1} + \gamma[by_{t-1} + (1-b)y_{t-1}] + \epsilon_t \\ &= x_{t-1} + \gamma y_{t-1} + \epsilon_t \\ &= x_{t-1} + \gamma(g-1)0.5(x_{t-1} + x_{t-2}) + (\epsilon_t + \gamma v_{t-1}) \\ &= \phi_1 x_{t-1} + \phi_2 x_{t-2} + [\epsilon_t + \gamma v_{t-1}], \end{aligned}$$

where $\phi_1 = 1 + 0.5\gamma(g-1)$ and $\phi_2 = 0.5\gamma(g-1)$. When expectations are naive, prices follow an AR(2) process that depends on current supply shocks and one-period lagged demand shocks.

(e) (10 pts) Use the implied AD-AS diagram for the model to discuss the Phillips curve tradeoff faced by policymakers.

Solution. If g is high and the monetary authorities accomodate price level increases, then the inverse demand curve is steep and supply shocks will lead to large fluctuations in prices but relatively stable output. On the other hand, if g is low and the monetary authorities do not accomodate price level increases, then the inverse demand curve will be shallow and supply shocks will lead to larger fluctuations in output but more stable prices. Therefore, through the choice of g, the central bank faces a tradeoff. They can either choose price-level stability or output stability, but not both.

2. Dynamic New Keynesian Model and the Taylor Principle. (50 pts) Write down the canonical three-equation Dynamic New Keynesian (DNK) model with a brief explanation of each equation. Using an AD-AS diagram, describe intuitively how the DNK model explains macroeconomic fluctuations. Also, describe the *Taylor principle* and the regime shift in monetary policy over the last 40 years. How does the Taylor principle relate to macroeconomic instability and aggregate fluctuations?

Solution. The DNK model can be written as follows:

$$\begin{aligned} x_t &= -\varphi[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + \epsilon_t \\ \pi_t &= \lambda x_t + \gamma E_t \pi_{t+1} + \mu_t \\ i_t &= i + \theta_x E_t x_{t+1} + \theta_\pi E_t \pi_{t+1}, \end{aligned}$$

where x_t is the output gap, π_t is inflation and i_t is the nominal interest rate. The first equation is the

IS curve, which states that the output gap is (negatively) related to the real interest rate, (positively) related to expected future output and a demand-side shock. The second equation is the Phillips curve, which states that inflation is (positively) related to the output gap, expected future inflation and a supply-side shock. The third equation is an interest rate rule, which relates the nominal interest rate to expected future output and inflation.

The Taylor principle says that $\theta_\pi > 1$, so that the monetary authorities raise nominal interest rates more than one-to-one with expected future inflation. This ensures that when inflation is expected to be high in the future, real interest rates will rise and dampen future inflation. Prior to the Volcker-Greenspan era, estimates of θ_π were less than one implying that expected future inflation could be self-fulfilling and contribute to macroeconomic stability and fluctuations.