ECON 5110 Class Notes
Topics in Real Business Cycle Theory

1 Introduction

In this section, we examine several well-known shortcomings and modifications of RBC theory. The topics, while far from exhaustive, give a general sense of the direction that RBC research has taken since the seminal works of Kydland and Prescott (1982) and Long and Plosser (1983). We focus first on the labor market.

2 Labor Market

2.1 Hansen (1985): Labor Indivisibilities

A primary failure of standard RBC models is that the total number of hours worked do not fluctuate enough to match the U.S. data. To match the volatility, the intertemporal elasticity of labor supply must be increased to a level that is inconsistent with panel micro studies.

Hansen (1985) addresses this failure by modeling the extensive margin of labor supply (i.e., movement in and out of the labor market). Standard RBC models only model the intensive margin (i.e., variation in number of hours worked per week). Total hours worked (H_t) can be decomposed according to

$$var(\log H_t) = var(\log h_t) + var(\log N_t) + 2cov(\log h_t, \log N_t).$$

The first term is the intensive margin and accounts for approximately 20% of labor market volatility. The second term is the extensive margin and accounts for 55% of labor market volatility. The remainder is captured by the covariance term.

In Hansen's model, there is substantial variation in total labor hours as individuals move in and out of the labor market. This occurs even though individuals have a low intertemporal elasticity of labor supply.

2.1.1 Model with Divisible Labor

A continuum of identical, infinitely lived households have access to the production function

$$f(\lambda_t, k_t, h_t) = \lambda_t k_t^{\theta} h_t^{1-\theta} \tag{1}$$

where h_t is labor, k_t is the capital stock, and λ_t is a stochastic productivity shock that follows a first-order Markov process:

$$\lambda_{t+1} = \gamma\lambda_t + \varepsilon_{t+1}. \tag{2}$$

The driving shocks, ε_{t+1}, are i.i.d. and have distribution function F with mean $(1-\gamma)$ and positive support. The resource constraint is

$$c_t + i_t \leq f(\lambda_t, k_t, h_t), \tag{3}$$

where c_t is consumption and i_t is investment. The law of motion for capital is given by

$$k_{t+1} = (1-\delta)k_t + i_t, \tag{4}$$

where δ is the rate of depreciation.

Households choose a path for c_t and $l_t = 1 - h_t$ that maximize

$$E\sum_{t=0}^{\infty} \beta^t \left(\log c_t + A \log l_t\right) \tag{5}$$

subject to their individual budget constraint

$$c_t + i_t \leq w_t h_t + r_t k_t, \tag{6}$$

where w_t is the wage rate and r_t is the rental rate for capital.

This problem can be solved using *dynamic programming* methods and the Bellman equation:

$$V(k,\lambda) = \max\left\{u(c, 1-h) + \beta E[V(k', \lambda')|\lambda]\right\} \tag{7}$$

subject to the standard constraints. This problem does not have an analytical solution and requires numerical methods.

2.1.2 Model with Indivisible Labor

Households are restricted to either work full time $h_t = h_0$ or not at all $h_t = 0$. All variation in labor supply is along the extensive margin.

This would be a non-convex problem and require integer programming methods. To circumvent this issue, Hansen assumes that individuals choose a probability of working, α_t, and their employment status is determined by a lottery.

Households and firms sign a contract that requires a household to work h_0 hours with probability α_t. A fraction α_t of households will work. The firm offers complete unemployment insurance to workers so the $(1-\alpha_t)$ fraction of unemployed households will receive identical pay.

Expected utility is given by

$$\begin{aligned} E[u_t] &= \alpha_t \left[\log c_t + A\log(1-h_0)\right] + (1-\alpha_t)\left[\log c_t + A\log(1)\right] \\ &= \log c_t + A\alpha_t \log(1-h_0) \end{aligned} \tag{8}$$

with per capita hours worked given by

$$h_t = \alpha_t h_0. \tag{9}$$

Firms will employ labor to satisfy

$$f_h(\lambda_t, k_t, h_t) = w_t. \tag{10}$$

Households, however, are <u>not</u> paid for the time they spend working. They are paid for the <u>expected</u> time spent working (i.e., each worker is paid as if she works h_t). Households therefore have full unemployment insurance. The appendix shows that this is equivalent to having a market for unemployment insurance.

The representative household therefore chooses α_t, c_t, and i_t to maximize

$$E\sum_{t=0}^{\infty}\beta^t u(c_t, \alpha_t) \tag{11}$$

subject to

$$c_t + i_t \leq w_t\alpha_t h_0 + r_t k_t \tag{12}$$

and the law of motion for capital. This is a standard concave dynamic programming problem that can be solved using various methods. The state variables are λ_t and k_t. The control variables are α_t, c_t, and i_t.

A key result is that the aggregate economy exhibits an infinite intertemporal elasticity of substitution (IES) for labor even though individual households have a low IES. To see this, substitute

$$l_t = 1 - h_t = 1 - \alpha_t h_0 \tag{13}$$

into the expected utility function (8):

$$u(c_t, \alpha_t) = \log c_t + A\alpha_t \log(1-h_0).$$

This produces

$$\begin{aligned} u(c_t, l_t) &= \log c_t + A[(1-l_t)/h_0]\log(1-h_0) \\ &= (A/h_0)\log(1-h_0) + \log c_t - (A/h_0)\log(1-h_0)l_t. \end{aligned} \tag{14}$$

The utility function for the representative agent can be rewritten more simply as

$$u(c_t, l_t) = \log c_t + Bl_t, \tag{15}$$

where $B = -(A/h_0)\log(1 - h_0)$. This shows that the IES for labor is infinite for the aggregate economy even though the IES is low for individual households.

2.1.3 Solution and Calibration

The solution method is similar to Kydland and Prescott (1982) and involves a quadratic approximation to the objective function, and if necessary, a linear approximation to the constraints. This creates a linear-quadratic (LQ) problem that can be solved with numerical methods.

The calibration is standard and results in the following parameter values:

Parameter	β	A	θ	δ	h_0	γ	σ_ε
Value	0.99	2	0.36	0.025	0.53	0.95	0.00712

2.1.4 Simulation Results

The U.S. and artificial data are logged and passed through the HP filter. The simulations involve 100 replications; each replication contains 115 observations. Table 1 shows the standard deviations and correlations (with output) for the U.S. data and the means from the simulations.

The main results are...

- The volatility of the indivisible labor economy is greater than the divisible labor economy.
- The ratio of the variability of hours worked to productivity (2.7) is now higher than in the U.S. data (1.4). The ratio for the divisible economy is 1.17.

2.1.5 Conclusions

- Hansen introduces a model where households enter and exit employment based on productivity shocks.
- The model captures the fact that the majority of variation in total U.S. hours worked is along the extensive margin.
- The indivisible labor model exhibits an infinite intertemporal elasticity of substitution while being consistent with households reluctance to vary their labor supply over time.
- To match the volatility of U.S. total hours worked, it is necessary to use a model with variation in employment (extensive margin) and hours worked per week (intensive margin).

2.2 Christiano and Eichenbaum (1992): Government Spending

This paper addresses another shortcoming of the standard RBC model. In the U.S. data, the correlation between hours worked (n_t) and the return to working (y_t/n_t) are weakly correlated (so called Dunlop-Tarshis observation). In the RBC model, the correlation is strong and positive. Equilibrium RBC hours worked come from the intersection of a static labor supply curve and a stochastic labor demand curve. Therefore, hours worked and productivity have a strong positive correlation in the model. The U.S. data, on the other hand, are likely influenced by shifts in both labor demand and labor supply.

Christiano and Eichenbaum add government consumption shocks to the model to address the Dunlop-Tarshis observation. However, any modifications that shift labor supply, such as changes in tax rates or demographic changes to the labor force would suffice.

2.2.1 Model

A social planner chooses consumption and leisure streams $\{c_t, N - n_t\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln(c_t) + \gamma V(N - n_t) \right\} \tag{16}$$

where consumption

$$c_t = c_t^p + \alpha g_t \tag{17}$$

is comprised of endogenous private consumption (c_t^p) and exogenous public consumption (g_t).

The leisure function $V(\cdot)$ takes one of two forms:

$$\begin{aligned} V(N - n_t) &= \ln(N - n_t) \text{ "divisible labor"}, \\ V(N - n_t) &= N - n_t \quad \text{ "indivisible labor"}. \end{aligned}$$

The other constraints are standard. Output (y_t) is given by a Cobb-Douglas technology:

$$y_t = (z_t n_t)^{(1-\theta)} k_t^{\theta}. \tag{18}$$

Technology shocks follow

$$z_t = z_{t-1} \exp(\lambda_t). \tag{19}$$

The resource constraint is

$$c_t^p + g_t + k_{t+1} - (1 - \delta) k_t \leq y_t. \tag{20}$$

Solution To solve the model, it is necessary to normalize all relevant variables by the nonstationary technology shock z_t:

$$\begin{aligned} \bar{k}_{t+1} &= k_{t+1}/z_t \\ \bar{y}_t &= y_t/z_t \\ \bar{c}_t &= c_t/z_t \\ \bar{g}_t &= g_t/z_t. \end{aligned}$$

Normalized government consumption is assumed to follow:

$$\ln(\bar{g}_t) = (1-\rho)\ln(\bar{g}) + \rho \ln(\bar{g}_{t-1}) + \mu_t. \tag{21}$$

A solution to this problem requires numerical methods. Using a Taylor Series approximation, the decision rules can be written in the form:

$$\hat{k}_{t+1} = r_k \hat{k}_t + d_k \hat{g}_t + e_k \hat{\lambda}_t \tag{22}$$

and

$$\hat{n}_t = r_n \hat{k}_t + d_n \hat{g}_t + e_n \hat{\lambda}_t, \tag{23}$$

where hats (^) over the variables represent proportional deviations from steady state. The coefficients $(r_k, d_k, e_k, r_n, d_n, e_n)$ are interpreted as elasticities and functions of the structural parameters.

Comparative Static To understand the role of government consumption, consider the impacts of extreme values of α on d_n.

- When $\alpha = 1$, private and public consumption are perfect substitutes. Increases in g_t induce offsetting decreases in c_t^p. Therefore d_n (and d_k) are zero. Public consumption therefore has no role in the RBC model.

- When $\alpha = 0$, public consumption is a pure resource drain. Since leisure is a normal good, d_n is positive and labor supply shifts out in response to government spending shocks. The elasticity d_n is decreasing in α.

2.2.2 Econometric Methodology

Traditionally, RBC theorists calibrate their models using micro studies and historical averages. CE (1992) instead use Generalized Method of Moments (GMM) to estimate the structural parameters. This allows the uncertainty in the parameters to be quantified and the predictions of the model to be formally tested.

The methodology first estimates the structural parameters

$$\Psi_1 = \{\delta, \theta, \gamma, \rho, \bar{g}, \sigma_\mu, \lambda, \sigma_\lambda\} \tag{24}$$

using GMM methods. The estimate $\hat{\Psi}_1$ is then used to simulate artificial data and calculate simulated second moments

$$\Psi_2 = \{\sigma_{c^p}/\sigma_y, \sigma_{dk}/\sigma_y, \sigma_n, \sigma_n/\sigma_{y/n}, \sigma_g/\sigma_y, corr(y/n, n)\}. \tag{25}$$

These simulated and the empirical versions of Ψ_2 are then used to form

$$J = F(\hat{\Psi})' var\left[F(\hat{\Psi})\right]^{-1} F(\hat{\Psi}), \tag{26}$$

which is asymptotically chi-square.

2.2.3 Empirical Results

The main data series were taken from the U.S. Department of Commerce and the U.S. Bureau of Economic Analysis. The labor market data were taken from two different surveys administered by the U.S. Department of Labor Statistics: the Household Survey and the Establishment Survey. The sample period is 1955:3 through 1983:4.

- Table 1 shows the parameter estimates and standard errors using GMM.
- Table 3 shows the second-moment properties of HP-filtered simulated and actual U.S. data using the Household Survey.
- Table 4 shows the test statistics for contrasting the model and data.

2.2.4 Conclusions

The main conclusions from the CE (1992) article are....

- Adding government consumption to the baseline RBC model can help address the Dunlop-Tarshis observation by shifting labor supply.
- Methods for estimating the structural parameters and formally testing the RBC model are presented.
- Once government spending is added, the RBC model with indivisible labor cannot be rejected using the Establishment Survey.
- Allowing for measurement error, the RBC model with indivisible labor cannot be rejected using either the Establishment or Household Survey.

2.3 Aadland (2001): High-Frequency Real Business Cycles

This paper attempts to explain the same two labor-market puzzles addressed in Hansen (1985) and CE (1992):

- volatility of hours worked relative to output and
- correlation of real wage and hours worked.

While those papers added new features (i.e., labor indivisibilities and government spending) to the baseline RBC theory, this paper maintains the baseline theory and simply changes the decision interval. Nearly all RBC models assume that agents make decisions at a quarterly frequency (i.e., four times per year). This assumption is made so that the artificial data from the model can be easily compared with the U.S. quarterly data. However, agents likely make decisions more frequently than four times per year. In this paper, I allow agents to make decisions more frequently (e.g., once per week), temporally aggregate the artificial data up to the quarterly level, and then reassess the performance of the RBC model.

2.3.1 Benchmark Model

A representative agent is assumed to choose weekly consumption $\{c_t\}$ and hours worked $\{n_t\}$ streams to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j [(1-\phi)\log(c_{t+j}) + \phi \log(N - n_{t+j})]$$

subject to the resource constraint

$$k_t^{\theta}(a_t n_t)^{1-\theta} \geq c_t + k_{t+1} - (1-\delta)k_t$$

and the law of motion for technological change (a_t)

$$a_t = a_{t-1}\exp(\mu + \epsilon_{at}).$$

2.3.2 High-Frequency Calibration

The weekly model is calibrated by adjusting the standard quarterly parameter values using ad hoc transformation rules. Denoting the weekly parameters with asterisks, for example

- $\beta_* = \beta^{1/13} = 0.9926^{1/13} = 0.9994.$
- $N_* = N/13 = 1369/13 = 105.3.$

- $\phi_* = \phi = 2/3$.

See Table 1 for the full set of calibrated parameters.

2.3.3 Simulation and Temporal Aggregation

The calibrated RBC model is then solved and used to simulate weekly artificial data. The weekly data are then aggregated over time up to the quarterly frequency using the same procedures employed by the U.S. data collection agencies (e.g., Bureau of Economic Analysis, Bureau of Labor Statistics). A particularly important fact is that the household and establishment surveys administered by the BLS use the "calendar week that contains the 12th day of the month" as the reference period for the entire month.

The temporal aggregation operators are

$$B_{TA}(L) = 1 + L + L^2 + \cdots + L^{12}$$

for flow variables such as output and consumption and

$$B_{SS}(L) = \frac{13}{3}(L^2 + L^6 + L^{10})$$

for flow variables such as total hours worked, where L is the lag operator satisfying $L^j X_t = X_{t-j}$. The aggregation operators are more complex when the variables are measured in logarithms and first differences.

2.3.4 Comparison of the Basic and Aggregate Covariances

Since RBC theorists focus on the second-moment properties of the data (i.e., standard deviations and cross correlations), it will be useful to relate the aggregate (quarterly) and basic (weekly) covariances. The general relationship is

$$\Gamma_{xy}(0) = \omega_x \gamma_{xy}(0) \omega_y' \tag{27}$$

where

- $\Gamma_{xy}(0)$ is the zeroth-order covariance between aggregate x and aggregate y;
- ω_x is the vector describing the temporal aggregation of x;
- ω_y is the vector describing the temporal aggregation of y;
- $\gamma_{xy}(0)$ is the matrix of autocovariances between basic x and basic y.

For example, consider three-period aggregation ($n = 3$) of a flow variable x with $B_{TA}(L) = 1 + L + L^2$ and end-of-period sampling of a stock variable y with $B_{SS}(L) = 1$. Using equation (27) and assuming the variables are measured in growth rates, we get

$$\Gamma_{xy}(0) = \frac{1}{3}(\gamma_{-4} + 3\gamma_{-3} + 6\gamma_{-2} + 7\gamma_{-1} + 6\gamma_0 + 3\gamma_1 + \gamma_2).$$

Continuing with this example, imagine that x is output and y is the real wage. The RBC model predicts a large positive contemporaneous covariance, say, $\gamma_0 = 0.9$, and weaker cross covariances, say, $\gamma_j = -0.25$ for $j = -4, -3. -2, -1, 1, 2$. So even if the RBC theory were the true model, we would observe, after aggregation

$$\Gamma_{xy}(0) = \frac{1}{3}[-21(0.25) + 6(0.9)] = 0.05$$

in the U.S. data and approximately $\Gamma_{xy}(0) = 0.9$ from the standard quarterly RBC model. Therefore, we would mistakenly reject the RBC theory.

2.3.5 Results

For the complete set of results, see Table 2 on page 285. Here is a summary of the main results

Statistic	U.S. Data	Baseline RBC Model		Home Production Model	
		Quarterly	Weekly	Quarterly	Weekly
std(y)	0.96	0.96	0.96	0.96	0.96
std(n)	0.94	0.30	0.38	0.67	0.88***
corr(w,n)	-0.35	0.97	0.60	0.12	-0.30***

where (***) indicates failure to reject that the statistic is equal to that in the U.S. data. The home production model improves the performance of labor-market fluctuations because home-production technology shocks generate additional substitution between market and home activities, which also weakens the correlation between market wages and hours worked.

2.3.6 Conclusions

A strong assumption in RBC models (as well as other models of the business cycle) is that agents make decisions once per quarter. In reality, agents in our economy make economic decisions on a much more frequent basis. This paper modifies a standard RBC model to allow agents to make decisions on a weekly basis. With careful treatment of the time aggregation and sampling properties of actual U.S. data, the weekly RBC model comes closer to resolving a couple of well-known labor-market anomalies. In fact, the weekly RBC model with household production is statistically indistinguishable from the U.S. economy along these two dimensions.