ECON 5110 Class Notes
Topics in Real Business Cycle Theory

1 Propagation

1.1 Cogley and Nason (1995): Output Dynamics

Cogley and Nason (1995) ask whether standard RBC models can match the stylized facts about output dynamics. Two well-known facts about U.S. output are

1. GNP growth is positively autocorrelated over short horizons and
2. GNP has a strong hump-shaped trend-reverting component.

See Figure 1, page 494.

1.1.1 Baseline RBC Model

Cogley and Nason use the CE (1992) RBC model as their baseline case. A representative agent maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j [\log(c_{t+j}) + \gamma(N - n_{t+j})]$$

subject to the resource constraint

$$y_t = k_t^{\theta}(a_t n_t)^{1-\theta} \geq c_t + k_{t+1} - (1-\delta)k_t$$

and two shocks. The technology shock follows

$$a_t = a_{t-1} \exp(\mu + \epsilon_{at})$$

and the government spending shock follows

$$\bar{g}_t = g \times \bar{g}_{t-1}^{\rho} \exp(\epsilon_{gt})$$

where $g = \exp((1-\rho)\bar{g})$ and $\bar{g}_t = g_t/a_t$. The model is calibrated as follows:

Parameter	β	γ	θ	δ	μ	$\bar{g}$	ρ	σ_a	σ_g
Value	$1.03^{-0.25}$	0.0037	0.344	0.021	0.004	0.177	0.96	0.0097	0.0113

Using Monte Carlo simulations, 1000 artificial samples are generated, each with a length of 140 quarters.

1.1.2 Autocorrelation Function (ACF) Results

Using the generalized Q statistic with an asymptotic chi-square distribution,

$$Q_{ACF} = (\hat{c} - c)' \hat{V}_c^{-1} (\hat{c} - c),$$

where $\hat{c}$ is the vector of U.S. autocorrelations, $c = \frac{1}{1000} \sum_{i=1}^{1000} c_i$, and $\hat{V}_c = \frac{1}{1000} \sum_{i=1}^{1000} (c_i - c)(c_i - c)'$, Cogley and Nason test whether the CE RBC and the U.S. output growth autocorrelations are different. CN soundly reject the null that they are the same ACFs (see Table 1 and Figure 3). In fact, the autocorrelation function in the CE RBC model is very nearly zero at all horizons.

1.1.3 Impulse Response Function (IRF) Results

Using the same generalized Q test, CN also reject that transitory and permanent IRFs from the CE RBC model are equal to those in the U.S. data (see Table 1 and Figure 3). In particular, the CE RBC model produces a damped, hump-less response to transitory shocks.

1.1.4 Propagation

In an earlier paper in *Economics Letters*, Cogley and Nason (1993) show that output in the CE RBC model can be decomposed into its permanent and transitory parts: $y(t) = y_P(t) + y_T(t)$ where

$$y_P(t) = \left[\frac{\epsilon_a(t)}{1 - L} \right] \left[(1.04) \frac{1 - 0.87L}{1 - 0.94L} \right]$$

and

$$y_T(t) = \left[\frac{\epsilon_g(t)}{1 - \rho L} \right] (0.16).$$

The first part of each expression refers to the shock dynamics and the second, the propagation dynamics. Clearly there is no propagation mechanism for the transitory part (i.e., dampening scalar 0.16) and very little with the permanent part since $(1 - 0.87L)$ and $(1 - 0.94L)$ nearly cancel out. Overall, this implies that output in the CE RBC model inherits the dynamics of the exogenous shock processes with little additional dynamics coming from within the model. This can also be seen in the ACFs because CE RBC output growth and the growth in total factor productivity are each white noise.

1.1.5 Employment Lags and Labor Adjustment Costs

Some progress on output dynamics can be made by incorporating employment lags (e.g., labor-hoarding model of Burnside et al., 1993 in which firms choose the number of workers before observing the state of the economy and then vary work effort afterwards) and adjustment costs to changing employment. Both models

do a better job of matching U.S. output dynamics, however they still require implausibly large transitory government spending shocks to match the magnitude of output dynamics.

1.1.6 Conclusions

The main contribution of Cogley and Nason (1995) is to document that standard RBC models are incapable of matching the output dynamics of U.S. GNP. This is primarily due to the absence of any endogenous propagation mechanism in the models. Therefore, output in the model basically inherits the dynamic properties of the exogenous shock processes.