ECON 5110 Class Notes
Topics in Real Business Cycle Theory

1 International

1.1 Backus, Kehoe and Kydland (1992): International Real Business Cycles

- This article investigates whether an open-economy version of the RBC model is consistent with the international data on output, consumption and trade.
- In closed economies, domestic savings (S) equals domestic investment (I).
- It is helpful to keep in mind the national income accounting identity: $NX \equiv EX - IM = S - I = (Y - C) - I$.
- In the world economy, countries experience correlated technology shocks. Given the ability to borrow and lend internationally, we should expect different co-movements of macro variables within and across countries.
- In particular, we might expect countries to have smoother consumption (as they share risk) and more volatile investment (as capital moves to find the highest return).

1.1.1 Properties of International Business Cycles

Table 1 shows the cyclical properties of HP-filtered U.S. quarterly data (1954-1989).

- Typical volatilities and co-movements.
- Net exports (NX) are less volatile than output and slightly counter-cyclical.

Table 2 shows the international co-movements for 12 developed countries.

- Contemporaneous correlations with the U.S. are generally positive for output and consumption.
- Contemporaneous correlations with the U.S. are larger for output than for consumption.
- Correlation between saving and investment rates vary widely across countries. The correlation is large and positive for Germany, Japan and the U.S.
- Net exports are negatively correlated with output.

1.1.2 Model of the World Economy

Two countries with the same preference/technology structure and a domestic labor pool. Each country produces the same good and is subjected to a specific technology shock.

The expected utility function is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^i, l_t^i)$$

where $U(c,l) = (c^\mu l^{1-\mu})^\gamma/\gamma$ and $i = \{h, f\}$.

Output in each country (y_t^i) is given by the production function:

$$F(\lambda, k, n) = \lambda k^\theta n^{1-\theta}.$$

World output $(y_t^h + y_t^f)$ is allocated to consumption and investment:

$$F(\lambda_t^h, k_t^h, n_t^h) + F(\lambda_t^f, k_t^f, n_t^f) = [c_t^h + c_t^f] + [x_t^h + x_t^f].$$

Net exports (nx_t^i) are given by

$$nx_t^i = y_t^i - (c_t^i + x_t^i).$$

The model also includes (i) distributed lag of leisure, (ii) inventories, and (iii) time-to-build technology. These three features are suppressed here.

Technology Shock Process The vector of technology shocks $\lambda_t = (\lambda_t^h, \lambda_t^f)$ follows a VAR(1) process:

$$\lambda_{t+1} = A\lambda_t + \epsilon_{t+1}$$

where $\epsilon_t = (\epsilon_t^h, \epsilon_t^f)$ is the vector of driving shocks. The driving shocks are serially independent with variance-covariance matrix

$$V = \begin{bmatrix} \sigma_h^2 & \sigma_{h,f} \\ \sigma_{f,h} & \sigma_f^2 \end{bmatrix}.$$

The technology shocks have

- contemporaneous correlation $\sigma_{h,f} = \sigma_{f,h}$ and
- spillover effects A_{12} and A_{21}.

Welfare Theorems and Solution Technique The social planner problem is to maximize

$$\psi E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^h, l_t^h) + (1-\psi) E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^f, l_t^f)$$

for $\psi = 0.5$. The second welfare theorem of economics states that this solution can be supported as a competitive equilibrium for a certain set of prices. The first welfare theorem states that this competitive equilibrium is Pareto optimal.

Backus et al.'s solution technique involves...

- Substitute the constraints into the objective.
- Approximate the resulting function near the steady state using a second-order Taylor series approximation.

1.1.3 Steady State and Parameter Values

The model is calibrated for symmetric countries, except for A and V. The steady state for the world economy is therefore the closed economy replicated twice. The parameter values are

Parameter	β	c/y	x/y	δ	θ	σ	J	μ	γ
Value	0.99	0.75	0.25	0.025	0.36	0.01	4	0.34	-1

The shock process is estimated from a bivariate VAR. For U.S./Europe it is

$$A = \begin{bmatrix} 0.904 & 0.502 \\ 0.149 & 0.908 \end{bmatrix}$$

with $\sigma_{\epsilon,h} = 0.00906$, $\sigma_{\epsilon,f} = 0.00797$, and $corr(\epsilon_t^h, \epsilon_t^f) = 0.258$. The eigenvalues are 0.994 and 0.818.

For U.S./Canada it is

$$A = \begin{bmatrix} 0.796 & 0.131 \\ 0.000 & 0.989 \end{bmatrix}$$

with $\sigma_{\epsilon,h} = 0.00874$, $\sigma_{\epsilon,f} = 0.01023$, and $corr(\epsilon_t^h, \epsilon_t^f) = 0.434$. The eigenvalues are 0.989 and 0.796.

The benchmark, symmetrized version is

$$A = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix}$$

with $\sigma_{\epsilon,h} = \sigma_{\epsilon,f} = 0.00852$ and $corr(\epsilon_t^h, \epsilon_t^f) = 0.258$.

1.1.4 Results

Table 4 shows the results from the benchmark theoretical world economy.

- Standard deviation of simulated output is 1.55% (U.S. economy is 1.71%).
- Standard deviation of simulated consumption is 0.62% (U.S. economy is 0.84%).
- Standard deviation of simulated investment is 16.91% (U.S. economy is 5.38%).
- Standard deviation of simulated nx/y ratio is 2.90% (U.S. economy is 0.45%).
- Correlation of simulated nx/y and y is -0.02 (U.S. correlation is -0.36).
- Correlation of simulated saving and investment rates is 0.28 (U.S. correlation is 0.68).
- Correlation of simulated home and foreign output is -0.18 (U.S./Europe correlation is 0.70).
- Correlation of simulated home and foreign consumption is 0.88 (U.S./Europe correlation is 0.46).

Figure 2 presents the IRFs for home and foreign technology shocks from the benchmark economy. Figure 2 presents the intuition behind the co-movements of the home and foreign variables.

Backus et al. (1992) also consider other variations of the model: (*i*) asymmetric spillovers; (*ii*) large spillovers; (*iii*) high risk aversion; (*iv*) durable leisure; (*v*) one-quarter time-to-build; (*vi*) transport costs; and (*vii*) autarky.

1.1.5 Conclusions

- Backus et al. (1992) investigate how the standard RBC performs in a symmetric two-country global model.
- Backus et al. find a robust consumption/output anomaly:
  - Consumption across countries is more highly correlated in the model than in the data.
  - Output across countries is more highly correlated in the data than in the model.