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Rules Rather than Discretion: The Inconsistency of Optimal Plans
Kydland and Prescott (1977)

1. Introduction

Most of us have taken or are currently taking some sort of optimal control theory class. At the very least we have seen basic applications in advanced macroeconomic models. The basic idea is to maximize a (social) objective function over a period of time. Finn E Kyland and Edward C Prescott (KP) put forward the idea that when dealing with "rational economic agents", optimal control theory yields a sub-optimal outcome. That is, a decision rule that is time consistent does not necessarily maximize the objective function.

2. The Basics

2.1. Key Terms and Ideas.

In order to wrap our heads around this concept we need to define or at least clarify a few key concepts rooted in this paper. KP use the framework of some sort of objective function, subject to a set of decision-forming constraints. The social planner picks a policy in order to (hopefully) optimize the objective function subject to the decisions of the economic agents.

Optimal. A policy is optimal if it maximizes, over a given period of time, some objective function given a set a of constraints.

Consistent. A policy is consistent if, for each time period, the policy optimizes the objective function, *taking as given the set of previous decisions* and that future policy decisions are similarly selected.

Policy Rule. A policy rule is one that is "written in stone". That is, the policy is agreed upon and the implementation of which does not depend on current conditions.

Discretionary Policy. A discretionary policy is one that would factor current conditions into a general framework.

2.2. An Example.

Confused yet? Yeah, we know - it took us quite a few reads to wrap out head around these concepts. Here is a simple example to illustrate these ideas:

Let us suppose that there are developers looking into building in a well-known gas field in Sublette county. We can assume that given the effects of hydraulic fracturing on ground water quality, it is socially optimal to not build in these areas. The State of Wyoming could publicly pronounce that moving forward, they would not be responsible for providing any water or environmental protection, regardless of circumstance for any new development. This would be a policy rule - written in stone. Alternatively, the State could come up with a general policy that would provide support for the citizens of Wyoming within reason. This would be an example of a discretionary policy - a general framework with room for other factors to be considered.

Now let's assume that the economists at the University of Wyoming do a cost-benefit analysis and conclude that it is socially optimal for there to be no development in the gas fields. If the state enacted the "policy rule", the developers would know there could be problems with environmental conditions and rational agents would not move to the gas fields without protection. However, if the state were to have discretionary policy, the (rational) developers and new residents would know that if they moved to the fields, the government would/could step in and start trucking in water (i.e. Pavilion). Thus, the *optimal* decision would be to create a hard and fast *rule*, preventing rational agents from developing in the gas fields. However, with a *discretionary policy*, the agents (builders, homeowners) form rational expectations of the future policy decisions and build in the fields. Then, the *consistent* decision is to provide drinking water, which is suboptimal.

Other examples could be:

- floodplains
- taking the comps
- bank bailouts
- Cournot duopoly (?)

Add-in Consistent Policy. Let $\pi = (\pi_1, \pi_2, ..., \pi_T)$ be a sequence of policies for periods 1 to T (which may be infinite) and $x = (x_1, x_2, ..., x_T)$ be the corresponding sequence for econommics agents' decisions. The following agreed-upon social objective function is assumed to exist:

$$S(x_1, ...x_T, \pi_1, ..., \pi_T) \tag{1}$$

Further, agents' decisions in period t depend upon all policy decisions and their past decisions as follows:

$$x_t = X_t(x_1, ..., x_{t-1}, \pi_1, ..., \pi_T), t = 1, ..., T. \tag{2}$$

In such a framework an optimal policy, if it exists, is that feasible π which maximizes (1) subject to constraints (2).
Again, the definition of a consistent policy is:
A policy is *consistent* if, for each time period, t, π_t maximizes (1), taking as given previous decisions, $x_1, ..., x_{t-1}$, and that future policy decisions (π_s for $s > t$) are similarly selected.

2.3. **A Simple Mathematical Example.** This is a simple two-period (discrete time) model that KP use to motivate their argument. Let $T = 2$ and π_2 is a policy selected (in period $t = 2$) that is to maximize some generic social objective function

$$S(x_1, x_2, \pi_1, \pi_2), \tag{3}$$

subject to

$$x_1 = X_1(\pi_1, \pi_2)$$

and

$$x_2 = X_2(x_1, \pi_1, \pi_2). \tag{4}$$

The social planner chooses some policy, π that maximizes the objective function, knowing that the economic agents will make decisions, x based upon all past decisions and all past *and future policy* decision. When dealing with future policy decisions, we assume that the agents will use some form of rational expectations.

So, for a plan to be consistent, it must maximize (3), given the past decisions π_1, x_1 and (4). (Assuming differentiability and an interior solution) The FONC is

$$\underbrace{\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial \pi_2}}_{\text{“A”}} + \underbrace{\frac{\partial S}{\partial \pi_2}}_{\text{“B”}} = 0.$$

Here, **A** is the indirect effect of the policy on the objective function via the agents' decision in period 2. **B** is the direct of the policy on the objective function. Notice though that the policy maker takes x_1 as given. In this sterile mathematical world, technically it is. But in reality, x_1 is a function of the agents' *expectation* of π_2. (The consistent policy ignores the effects of π_2 upon x_1).

Now if the policy maker were to take into account this fact, the optimal decision rule would be

$$\underbrace{\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial \pi_2}}_{\text{“A”}}+\underbrace{\frac{\partial S}{\partial \pi_2}}_{\text{“B”}}+\underbrace{\frac{\partial S}{\partial x_1}\frac{\partial X_1}{\partial \pi_2}}_{\text{“C”}}+\underbrace{\frac{\partial S}{\partial x_2}\frac{\partial X_2}{\partial x_1}\frac{\partial X_1}{\partial \pi_2}}_{\text{“D”}}=0.$$

Only if **C** plus **D** is equal to zero, is the consistent policy also optimal. KP sum this up with the following statement:

"There is no mechanism to induce *future* policymakers to take into consideration the effect of their policy, via the expectations mechanism, upon *current* decisions of agents."

2.4. **Phillips Curve Example.** Another example KP put forth is a graphical one relating to inflation and unemployment. Let us assume that there is some social objective function

$$S(x_t, u_t), \tag{5}$$

where x_t is inflation and u_t is unemployment. Standard macroeconomic theory links these two variables via the Phillips curve,

$$\begin{aligned} u_t &= \lambda(x_t^e - x_t) + u^* \\ \Rightarrow x_t &= x_t^e - \lambda^{-1}(u_t - u^*), \end{aligned} \tag{6}$$

where λ is some positive constant, x_t^e is the expected inflation rate and u^* is the natural rate of unemployment. KP assume that the optimal rate of inflation and unemployment (or deviation from the natural rate) are both zero. See figure 1 for a visualization of this problem.

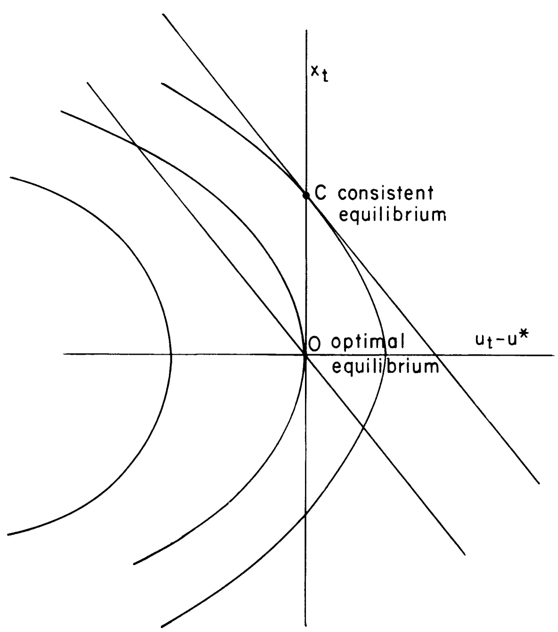

FIG. 1.—Consistent and optimal equilibrium

Thus, by maximizing (5) subject to (6), the consistent solution is along the indifference curve lying tangent to the Phillips curve, above at point "C". Visually, this is clearly not optimal. If the policymakers were forced to maintain price stability and did not have discretionary powers, the resulting equilibrium would be at point "O". Here, there would be the same unemployment and lower inflation. That is, if the optimization is not constrained by the agents' expectations (in the Phillips Curve), the policymakers could reach a higher social indifference curve.

(for this structure considered, such policy results in excessive rates of inflation without any reduction in unemployment.)

3. add-in Consistent Planning for the Infinite Horizon

The method of backward induction cannot be applied to infinite-period problems to determine a consistent policy because,unlike the finite period problem, there is no final period with which to begin the induction.

If using recursive structures, the concept of consistency can be defined in terms of policy rules. Suppose that the economy at time t can be described by a vector of state variables y_t, a vector of policy variables π_t, a vector of decision variables x_t, for the economic agents, and a vector of random shocks ϵ_t. The movement over time of these variables is given by the system of equations

$$y_{t+1} = F(y_t, \pi_t, x_t, \epsilon_t)$$

Let the feedback policy rule for the future periods be

$$\pi_s = \Pi^f(y_s), s > t \tag{7}$$

For certain structures, rational economic agents will in the future follow a rule of the form,

$$x_s = d^f(y_s; \Pi^f). \tag{8}$$

We could see that changes in policy rule Π^f change the functional form of d^f. (Lucas paper, (1976)). The decisions of agents in the current period will have the form

$$x_t = d^c(y_t, \pi_t; \Pi^f). \tag{9}$$

If we have the social objective function as:

$$\Sigma_{s=t}^{\infty} \beta_\pi^s q(x_s, y_s, \pi_s), 0 < \beta_\pi < 1, \tag{10}$$

we could see that π_t will depend upon both y_t and Π^f, which is the rule will be used in the future. In other words, the best policy rule for the current period $\Pi^c(y)$ is functionally related to the policy used in the future $\Pi^f(y)$, which would be written as

$$\Pi^c = g(\Pi^f) \tag{11}$$

A stationary policy rule Π is consistent if it is a fixed point of mapping g, for then it is best to use the same policy rule as the one expected to be used in the future. (please see Phelps and Pollak (1968) for this solution concept)

Suppose policymakers and agents do not have a clear understanding of the dynamic structure of the economy. Over time, agents will learn and most likely converge to the equilibrium rules of forms (7) and (8). After this, the procedure seems would run as before and is likely to converge to the consistent but suboptimal policy.

4. Conclusion

(It is hard to blame a policymaker acting consistently. The reason that such policies are suboptimal is not due to myopia. The effect of this decision upon the entire future is taken into consideration. Rather, the sub-optimality arises because there is no mechanism to induce induce future policymakers to take into consideration the effect of their policy, via the expectations mechanism, upon current decisions of agents.)

(Again, as a conclusion, KP paper argued that control theory is not the appropriate tool for dynamic economic planning. It is not the appropriate tool because current decisions of economic agents depend upon expected future policy, and these expectations are not invariant to the plans selected.)

(They have shown that, if in each period the policy decision selected is the one which maximizes the sum of the value of current outcomes and the discounted valuation of the end-of-period state, the policy selected will be consistent but not optimal,(please check the math procedure and derivation if you want)). In their final of the paper, they also find out that active stabilization effects did, for some distributed lag expenditure schedules, contribute to economic instability and even make a stable economy unstable.

4.1. **Policy suggestion.** KP argued that the structures used in this paper are far from a tested theory of economic fluctuations, and a tested theory is needed before policy evaluation is undertaken. The implication of this paper is that, until we have such a theory, active stabilization may very well be dangerous and it is best that it not be attempted. Reliance on policies such as a constant growth in the money supply and constant tax rates constitute a safer course of action.
They all suggest that when we do have the prerequisite understanding of the business cycle, the implication of this paper becomes that policymakers should follow rules rather than have discretion. The reason is that they should not have discretion is not that they are stupid or evil but, rather, that discretion implies selecting the decision which is best, given the current situation. Such behavior either results in consistent but suboptimal planning or in economic instability.
Finally, if we are not to attempt select policy optimally, how should it be selected? KP argued that as Lucas (1976) proposed, in a democratic society, it is probably preferable that selected rules be simple and easily understood, so it is obvious when a policymaker deviates from the policy. There could be institutional arrangements which make it a difficult and time-consuming process to change the policy rules, except in emergency situation. One possible institutional arrangement is for Congress to legislate monetary and fiscal policy rules and these rules to become effective only after a 2-year delay, and this would make discretionary policy impossible.

Optimal control theory is a powerful tool that has many applications in economics. However, when the agents use rational expectations, the time-consistent solution is not necessarily optimal. Using control theory, the policymaker will make an optimal choice, given the current situation and past decisions. However, knowing this, agents can rationally expect and make "suboptimal" decisions before the policymaker can react.