Recursive Residuals and Structural Change

Advanced Econometric Theory I

Pablo David Campoverde Varela
pcampove@uwyo.edu

December 16, 2019

UW | College of Business
Department
of Economics

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Introduction

What happens when an structural change takes place in our data?

- The model we are using is unreliable.
- The forecasting error becomes huge.

Introduction

When the break point is known we can use the dummy variable models learned in class.

- Intercept Dummies
- Slope Dummies
- Spline Regressions

But if we do not know the break point, we need a methodology to assess the stability of the parameters over time (constancy of the coefficients).

Recursive Parameter Estimation

- Recursive (or sequential) estimation can be thought of as arising as data points are sequentially brought into the computations.
- At each $t = k, ..., T-1$, we compute:

$$\hat{y}_{t+1} = \sum_{i=1}^{k} \hat{\beta}_{i,t} x_{i,t+1}$$

where t is all the observations used to estimate $\hat{\beta}$, then $\hat{y}_{t+1}$ is the estimation for the time $t+1$ using $\hat{\beta}_t$. And $i = 1, 2, ..., k$.

Recursive Residuals

- Now, we have the observation y_{t+1} and the estimation $\hat{y}_{t+1}$. The difference between them is the recursive residual:

$$\hat{e}_{t+1} = y_{t+1} - \hat{y}_{t+1}$$

- If the matrix $(X_t'X_t)$ is non-singular and we assume that that the error terms, are independent and normally distributed with means zero and variances σ^2, we know that

$$\hat{e}_{t+1} \sim N(0, \sigma^2 r_t)$$

where $r_t = 1 + x_{t+1}'(X_t'X_t)^{-1}x_{t+1}$

Recursive Residuals Variance

$$\hat{e}_{t+1} = y_{t+1} - \hat{y}_{t+1}$$

Replacing the right hand side

$$\hat{e}_{t+1} = x'_{t+1}\beta + \epsilon_{t+1} - x'_{t+1}\hat{\beta}_t$$

Taking the variance of the expression

$$var(\hat{e}_{t+1}) = var(\epsilon_{t+1} - x'_{t+1}(\hat{\beta}_t - \beta))$$

Because the errors and covariates are independent

$$var(\hat{e}_{t+1}) = var(\epsilon_{t+1}) + x'_{t+1}\sigma^2(X'_t X_t)^{-1}x'_{t+1}$$
$$var(\hat{e}_{t+1}) = \sigma^2(1 + x'_{t+1}(X'_t X_t)^{-1}x_{t+1}) = \sigma^2 r_t$$

Standardized Recursive Residuals

- To standardize the recursive residuals, we have

$$w_{t+1} \equiv \frac{\hat{e}_{t+1}}{\hat{\sigma}_t \sqrt{r_t}}$$

- f the matrix $(X_t' X_t)$ is non-singular and we assume that that the error terms, are independent and normally distributed with means zero and variances σ^2, then $w_{t+1} \sim i.i.d.\ N(0,1)$

Now we can test to see if they are constant over time with a certain confidence level

MatLab Code