ECON 5350 Midterm Exam – Fall 2023

Consider the following regression model without a constant

$$y_i = \beta_1 x_{1i} + \beta_2 x_{2i} + \epsilon_i;\ i = 1, ..., n$$

where the error terms are i.i.d., $\bar{x}_1 = \bar{x}_2 = 0$ and $corr(x_{1i}, x_{2i}) = 0$. Use this information to answer the following equally weighted questions.

1. Using summation notation, find the normal equations for the ordinary least squares (OLS) estimator of β_1 and β_2. Check the second-order conditions.

2. Solve the normal equations to find the OLS estimator of β_1 and β_2.

3. Write out the regression model in matrix notation. Clearly define all the matrices with appropriate dimensions.

4. Show the equivalence of the matrix-based OLS formula with the results from question #2.

5. Rework question #2 with $corr(x_{1i}, x_{2i}) = 1$ and the normalization $var(x_{1i}) = var(x_{2i}) = 1$. Comment on the results.

6. Find the pdf of each error term, $f_i(\epsilon_i)$, if the cumulative distribution is $F_i(\epsilon_i) = \frac{3}{4}\epsilon_i(1 - \frac{1}{3}\epsilon_i^2) + \frac{1}{2}$ with support $(-1, 1)$.

7. Find the mean (μ) and variance (σ^2) of the errors. Sketch the pdf of the errors to verify your answer.

8. What is the limiting and asymptotic distribution of the sample mean, $\bar{\epsilon}_n$.

9. Find the joint pdf for the errors, $f(\epsilon_1, ..., \epsilon_n)$.

10. Use $f(\epsilon_1, ..., \epsilon_n)$ to find the first-order condition for the maximum (log) likelihood estimate of β_1 and β_2. Is it possible to solve for β_1 and β_2? Comment.