

ECON 5350 Midterm Exam – Fall 2022

- 1. Probability Theory (30 pts).** Consider the following probability density function (pdf): $f(x, y) = 0.25; 0 \leq x, y \leq 2$ and zero elsewhere.
 - (a) Verify that $f(x, y)$ is a valid pdf.
 - (b) Find the marginal pdf for X , $f_x(x)$ and its cdf. Then use the results to find the mean, variance and moment generating function (MGF) for X .
 - (c) Find the pdf and mean of $Z = X^2$.
- 2. Statistics (30 pts).** Consider an i.i.d. random sample $\{X_1, X_2, \dots, X_n\}$ from the following pdf: $f(x) = \frac{1}{\theta}e^{-x/\theta}; x, \theta > 0$ and zero elsewhere.
 - (a) Find the mean and variance of X_i .
 - (b) Propose an estimator for θ using your random sample. Is your estimator unbiased? Defend your answer.
 - (c) Is the estimator you proposed in (b) a consistent estimator of θ ? Defend your answer.
- 3. Classical Linear Regression Model (40 pts).** Consider the following model: $Y_i = \beta_1 + \beta_2 X_i + \epsilon_i$ for $i = 1, \dots, n$.
 - (a) Calculate the OLS estimates for β_1 and β_2 using the sample data $Y = (1, 5, 6)$ and $X = (-1, 0, 1)$. Under what conditions are these estimates the minimum variance unbiased estimates?
 - (b) Graph the sample points and the estimated regression line. Denote the residuals and their values on the graph. Calculate the R^2 for this equation. Does this model provide a good fit? Explain.
 - (c) Now recalculate the OLS estimates for the regression model above with the additional variable $Z = (1, -2, 1)$, which is uncorrelated with X . Calculate the R^2 for the new regression model. Discuss the goodness of fit for this model using a graph with the new regression lines and Z fixed at each of its three values.
 - (d) Consider an hypothesis test for $\beta_2 = 1$. There are generally two types of errors that can be made when performing such an hypothesis test. Discuss the nature of the two errors and their relationship to one another. Use a graph of the test statistic's distribution to motivate your discussion.