ECON 5350 Final Exam – Fall 2023

Consider the following age-earnings model:

$$Wage_i = \beta_1 + \beta_2 Age_i + \epsilon_i \tag{1}$$

where $\epsilon_i \sim i.i.d.\ N(0, \sigma^2)$ and $i = 1, ..., N$. Wages are measured in dollars per hour, and age is measured in years.

1. (25 pts) **Estimation I**. Derive the ordinary least squares (OLS) estimator $b = (b_1, b_2)'$ in either matrix or summation form. Prove that it is an unbiased and consistent estimator of β, highlighting the necessary assumptions.

2. (25 pts) **Estimation II**. Assume you are given the sample of data $Wage = (10, 5, 18, 12, 15)$ and $Age = (40, 20, 60, 35, 45)$. Calculate the OLS estimates and the residuals. Plot the residuals against age. Is there any evidence of heteroscedasiticity? Assuming there is, propose an efficient estimation procedure.

3. (25 pts) **Hypothesis Testing**. Carefully describe three equivalent ways to test the hypothesis $\beta_2 = 0.5$ (there is no need to actually calculate the numerical value of the test statistic, just carefully describe the procedure). Assume the estimate of β_2 is negative. Does this imply that the null hypothesis will be rejected? Explain.

4. (25 pts) **Dummy Variables.** Consider two hypotheses: (a) a high school diploma leads to higher pay and (b) high school graduates progress up the pay scale faster than those without a high school diploma as they age. Modify equation (1) as necessary and describe the how to test each hypothesis.

5. (25 pts) **Spline Regression**. Derive the spline regression model with a knot at $Age = 40$.

6. (25 pts) **Marginal Effects**. Consider a modified version of equation (1) with a quadratic age-earnings profile. Describe a test procedure to test that $\frac{\partial wage}{\partial age} = 0.5$.

7. (50 pts) **Error Term Distribution**. Assume the error terms have a beta($\alpha = 2, \gamma = 1$) distribution:

$$f(\epsilon_t; c) = c\epsilon_t^{\alpha-1}(1 - \epsilon_t)^{\gamma-1},$$

where $0 \leq \epsilon_t \leq 1$.

(a) Find the constant c.

(b) How does the new error distribution change your answer in part (1)?

(c) How does the new error distribution change your answer in part (4)?

(d) Now assume $\alpha = 3$. Write down the natural log of the joint pdf of the errors.

(e) Write a paragraph discussing your preferred strategy for estimating β_1 and β_2 using the objective function in part (d).