

Chow Test

Bryana Funk & Collin DePaemelere

University of Wyoming

December 9, 2022

Testing Hypotheses

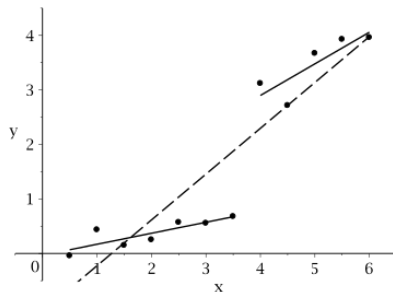
- F-Test: commonly used test for significance
- Primary test for linear regressions
- **What happens if there's a structural break?**

Outline

- Issues of F-Tests with structural breaks
- Chow Test Applications
- Chow Test Formula
- Real-data Example of Housing Purchases and Great Recession
- MATLAB

Possible Problems with the F-Test

- Structural breaks
 - Unexpected change over time in parameters
 - Caused by outside events (wars, pandemics, recessions, etc.)
 - Leads to forecasting errors



Applications of Chow Test

- Chow Test checks for a single break at time
- Break at $x = 1.7$
- Break doesn't line up for spline
- Two regressions gives a better model than combined

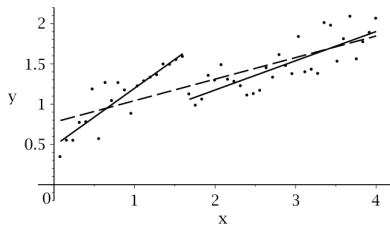


Figure: Structural break
(slopes differ)

Chow Test Formula

Suppose the simple model

$$Y = \alpha + \beta x_1 + \delta x_2 + \varepsilon$$

- α is the intercept
- β are δ are slope coefficients

Split data into two groups,

$$Y = \alpha_1 + \beta_1 x_1 + \delta_1 x_2 + \varepsilon$$

$$Y = \alpha_2 + \beta_2 x_1 + \delta_2 x_2 + \varepsilon$$

Chow Test Formula

Chow Test Hypothesis

- $H_0: \alpha_1 = \alpha_2, \beta_1 = \beta_2, \text{ and } \delta_1 = \delta_2$
- $H_A: \text{otherwise}$

Chow Test statistic

$$\begin{aligned} CHOW &= \frac{(SSE - (SSE_1 + SSE_2))/k}{(SSE_1 + SSE_2)/(N_1 + N_2 - 2k)} \\ &= \frac{(e'_* e_* - (e' e))/k}{(e' e)/(N_1 + N_2 - 2k)} \end{aligned}$$

where $CHOW \sim F(k, N_1 + N_2 - 2k)$

Chow Test Regression Formula

$$y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \dots + \beta_k x_{kt} + \gamma_0 D_t + \sum_{i=1}^k \gamma_i x_{it} D_t + \varepsilon$$

- D_t is a dummy variable taking a value of 1 for $i = n_1 + 1, \dots, n$ and 0 otherwise
- Chow Test Hypothesis
- $H_0: \gamma_0 = 0, \gamma_1 = 0, \dots, \gamma_k = 0$
- H_A : otherwise

Model of Pre- and Post-Recession Housing Purchases

- 2008 Great Recession is a large structural break
 - Lowered incomes from job loss
 - Mortgage market was very unstable
 - Economy wide changes pre- and post-recession
- Data Information
 - Aggregate U.S. quarterly data from 1/1/91 to 10/1/21 from FRED
 - U.S. quarterly Housing Price Index Federal housing Finance Agency (FHFA)

Restricted Model

$$\ln(HP)_t = \beta_0 + \beta_1 t + \beta_2 \ln(Displnc)_t + \beta_3 \ln(HPI)_t + \varepsilon_t$$

- HP_t - Housing Purchases
- t - Time of Observation
- $Displnc_t$ - Disposable Income
- HPI_t - Housing Price Index

Unrestricted Model

$$\ln(HP)_t = \beta_0 + \beta_1 t + \beta_2 \ln(Displnc)_t + \beta_3 \ln(HPI)_t + \beta_4 D_t + \\ \beta_5 (D_t \cdot t) + \beta_6 (D_t \cdot \ln(Displnc)_t) + \beta_7 (D_t \cdot \ln(HPI)_t) + \varepsilon_t$$

- D_t - Dummy: 0 Pre-2007, 1 2007
- $(D_t \cdot t)$ - Interaction of Dummy and t
- $(D_t \cdot \ln(Displnc)_t)$ - Interaction of Dummy and Disposable Income
- $(D_t \cdot \ln(HPI)_t)$ - Interaction of Dummy and Housing Price Index