Problem Set 8

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Final Project: Non-nested Tests

I - Definition and Background

Misspecification Test:

(1) **Nested Models:** One model that can be written as a special case of the other
  (a) $Y_i = \beta_1 + \beta_2 X_i + \epsilon_i$
  (b) $Y_i = \beta_1 + \epsilon_i$
  Model B is nested on A since $\beta_2 = 0$.
(2) **Non-nested Model:** One model cannot be written as a special case of the other
  (a) $Y_i = \beta_1 + \beta_2 X_i + \epsilon_i$
  (b) $Y_i = \alpha_1 + \alpha_e Z_i + \mu_i$

Tests Against Non-nested Alternatives

Example: Deciding whether an independent variable should appear in level or log form

$$Y_i = \beta_1 + \beta_2 X_{1i} + \beta_3 X_{2i} + \epsilon_i \quad \text{(Model 1)}$$
$$Y_i = \alpha_1 + \alpha_2 log(X_{1i}) + \alpha_3 log(X_{2i}) + \mu_i \quad \text{(Model 2)}$$

Two Approaches:

(1) J-test (Comprehensive Approach)
(2) Encompassing Model

J-Test

Developed by Davidson and Mackinnon (1981)

Example: Consider two models that are non-nested:

$$Y = \alpha_1 + \alpha_2 X_{1i} + \alpha_3 X_{2i} + \mu_i \quad \text{(Model A)}$$
$$Y = \beta_1 + \beta_2 X_{3i} + \beta_3 X_{4i} + \epsilon_i \quad \text{(Model B)}$$

Assume Model A is H_o.

In Particular:

H_o : Model A $(\alpha_4 = 0)$

H_a : Not Model A $(\alpha_4 \neq 0)$

We can use the estimated $\hat{Y}_b$ values from Model B as an additional regressor on Model A.

$$\hat{Y} = \hat{\alpha_1} + \hat{\alpha_2}X_{1i} + \hat{\alpha_3}X_{2i} + \hat{\alpha_4}\hat{Y}_b + \mu_i \tag{Model C}$$

We must check whether $\hat{\alpha_4}$ is statistically significant ($t - test$ or $p - value$):

- If $\hat{\alpha_4}$ is statistically significant then Model B > Model A
- If $\hat{\alpha_4}$ is not statistically significant then Model B < Model A

Answer Sample:

Model B is the preferred and "better" model over A.

Model A is the preferred and "better" model over B.

Example: When Model B is the reference model

Assume Model B is H_o.

In Particular:

H_o : Model B ($\beta_4 = 0$)

H_a : Not Model B ($\beta_4 \neq 0$)

We can use the estimated $\hat{Y}_a$ values from Model B as an additional regressor on Model A.

$$\hat{Y} = \hat{\beta_1} + \hat{\beta_2}X_{3i} + \hat{\beta_3}X_{4i} + \hat{\beta_4}\hat{Y}_a + \epsilon_i \tag{Model D}$$

We must check whether $\hat{\beta_4}$ is statistically significant ($t - test$ or $p - value$):

- If $\hat{\beta_4}$ is statistically significant then Model A > Model B
- If $\hat{\beta_4}$ is not statistically significant then Model A < Model B

Outcomes

When conducting a $J - test$ It is best to switch reference models to ensure which model is the better fit if any.

There are four possible outcomes of non-nested testing

Model A $\alpha_4 = 0$

		Fail to Reject	Reject
Model B $\beta_4 = 0$	Fail to Reject	Accept Both	Accept B and Reject A
	Reject	Accept A and Reject B	Reject Both

Two of which are not good outcomes.

Notes

Problems with Non-tested Testing

(1) Clear winners may not emerge among models
- Both models could be rejected
- Neither models could be rejected

(2) Even if H_o is rejected
- Unsure if model is correct

(3) If models have different dependent variables y vs. $log(y)$
- Non-nested testing does not apply to this situtation

Example 1

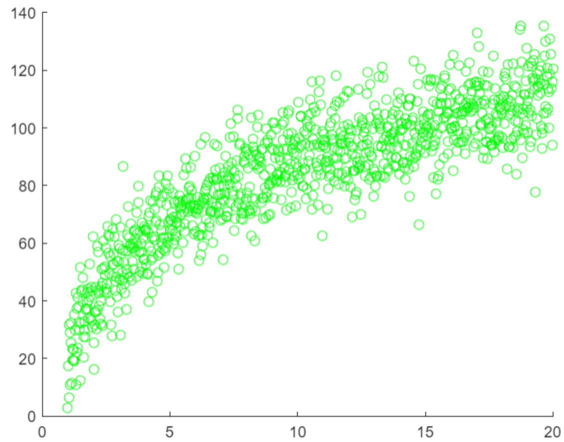

FIGURE 1. Model A: $Y = a_1 + a_2 \log(X_1) + u_i$

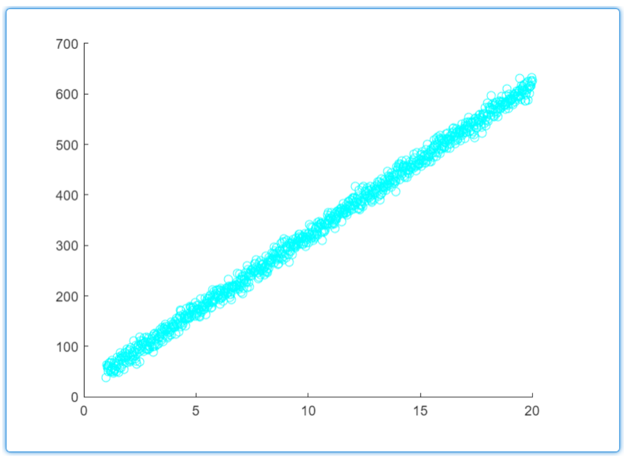

FIGURE 2. Model B: $Y = b_1 + b_2 X_1 + e_i$

Test whether Model A or Model B should be used to explain Y

Models:

- Model A: $Y = a_1 + a_2 log(X_1) + u_i$
- Model B: $Y = b_1 + b_2 X_1 + e_i$

Model A is your Reference Model

Model C is the Hybrid Model

- $\hat{Y} = \hat{a_1} + \hat{a_2} log(X_1) + \hat{a} Y_b + u_i$

Hypothesis

- H_o : Model A ($\alpha_4 = 0$)
- H_a : Not Model A ($\alpha_4 \neq 0$)

If α_4 is statistically significant, we reject Model A.

Example 2

Make Model B your Reference Model

Model D is the Hybrid Model

- $\hat{Y} = \hat{b_1} + \hat{b_2} X_1 + \hat{b} Y_a + e_i$

Hypothesis

- H_o : Model B ($\beta_4 = 0$)
- H_a : Not Model B ($\beta_4 \neq 0$)

If β_4 is statistically significant, we reject Model B.

ENCOMPASSING MODEL

Alternative Approach **Encompassing Model:** one in which the ability of one model to explain features another is tested Consider two non-nested models:

$$Y_i = \beta_1 + \beta_2 X_{1i} + \beta_3 X_{2i} + \epsilon_i \quad \text{(Model 1)}$$

$$Y_i = \alpha_1 + \alpha_2 log(X_{1i}) + \alpha_3 log(X_{2i}) + \mu_i \quad \text{(Model 2)}$$

Create a Hybrid Model:

$$\hat{Y} = \hat{\gamma}_1 + \hat{\gamma}_2 X_{1i} + \hat{\gamma}_3 X_{2i} + \hat{\gamma}_4 log(X_{1i}) + \hat{\gamma}_5 log(X_{2i}) + \epsilon_i \quad \text{(Model 3)}$$

We are distinguishing between H_o and the Hybrid Model

$H_o : \gamma_3 = 0$ and $\gamma_4 = 0$

H_o : Hybrid Model

- If $F - test$ is significant, reject H_o and infer that keeping the model in log form is important