ECON 5350 Problem Set #1

Due: Friday, September 22 by 11:59 pm

1. Consider the joint pdf in X_1 and X_2:

$$f(x_1, x_2) = 12x_1x_2(1 - x_2)$$

for $0 < x_1 < 1$, $0 < x_2 < 1$, and zero elsewhere. Use this joint pdf to answer the following questions:

   (a) Verify that $f(x_1, x_2)$ is a valid pdf.

   (b) Find the joint cdf for X_1 and X_2: $F(x_1, x_2)$.

   (c) Use MATLAB to graph the joint pdf and cdf.

   (d) Find the marginal pdfs for X_1 and X_2.

   (e) Verify that the marginal pdfs are valid.

   (f) Find the conditional pdfs for X_1 and X_2.

   (g) Are X_1 and X_2 stochastically independent? Defend your answer.

   (h) Find the moment generating function (MGF) for X_1 and X_2.

   (i) Use the MGFs to find the means and variances for X_1 and X_2.

2. Repeat all the steps in #1 for the joint pdf:

$$g(x_1, x_2) = 2e^{-x_1-x_2},$$

for $0 < x_1 < x_2$, $0 < x_2 < \infty$, and zero elsewhere.

   - In parts (h) & (i), just focus on X_1.
   - Perform a Monte Carlo simulation in MATLAB to plot the marginal pdf $g_1(x_1)$ and superimpose the continuous pdf.
   - Use the simulation to verify the mean and variance.

3. Use the change of variable technique to verify the log normal distribution.

4. Use the Gamma distribution and the following lifespan sample

$$w = (75, 78, 92, 64, 37, 91, 68, 82, 85, 45, 96, 72, 67)$$

to estimate the fundamental pdf parameters (α, β). Carefully describe your estimation technique.

5. Use MATLAB to graph the pdf and cdf for the number of wins for the Wyoming Cowboys' football season. Assume that they have a 0.60 probability of winning each remaining game. They need six wins to become bowl eligible. What are the chances this happens?

6. The all-time record for consecutive games in baseball where the same player gets a hit is held by Joe DiMaggio, professional baseball player for the New York Yankees in the 1940's. DiMaggio had at least one base hit in 56 continuous games, a streak thought by many to be nearly unbreakable. Make some plausible assumptions regarding batting percentage and number of attempts per game to calculate the probability of a 56-game hitting streak.