

# ECON 5350 Problem Set #1

Due: Thursday, September 20 at the beginning of class

## PAPER AND PENCIL PROBLEMS

1. Prove that the matrix formula for the least squares estimator,  $b = (X'X)^{-1}(X'Y)$ , is equivalent to

$$\begin{aligned} b_1 &= \bar{y} - \bar{x}_2 b_2 \\ b_2 &= \frac{\sum_{i=1}^n (x_{i,2} - \bar{x}_2)(y_i - \bar{y})}{\sum_{i=1}^n (x_{i,2} - \bar{x}_2)^2} \end{aligned}$$

when  $k = 2$ .

2. Prove that the matrix formula for the least squares estimator,  $b = (X'X)^{-1}(X'Y)$ , is equivalent to

$$\begin{aligned} b_1 &= \bar{y} - \bar{x}_2 b_2 - \bar{x}_3 b_3 \\ b_2 &= \frac{\sum_{i=1}^n (x_{i,2} - \bar{x}_2)(y_i - \bar{y})}{\sum_{i=1}^n (x_{i,2} - \bar{x}_2)^2} \\ b_3 &= \frac{\sum_{i=1}^n (x_{i,3} - \bar{x}_3)(y_i - \bar{y})}{\sum_{i=1}^n (x_{i,3} - \bar{x}_3)^2} \end{aligned}$$

when  $k = 3$  and  $\text{corr}(x_2, x_3) = 0$ .

3. Prove that the matrix formula for the variance of  $b$ ,  $\text{var}(b) = \sigma^2(X'X)^{-1}$ , produces:

$$\begin{aligned} \text{std.err.}(b_2) &= \frac{\sigma}{\sqrt{\sum_{i=1}^n (x_{i,2} - \bar{x}_2)^2}} \\ \text{std.err.}(b_3) &= \frac{\sigma}{\sqrt{\sum_{i=1}^n (x_{i,3} - \bar{x}_3)^2}} \end{aligned}$$

when  $k = 3$  and  $\text{corr}(x_2, x_3) = 0$ .

## MATLAB PROBLEMS

4. Use the state data in the file “resourcecourse.txt” and the regression model,  $Y = X\beta + \epsilon$ , to address the questions below. The sample period for economic growth is 1939-2004. The dependent variable is per capita personal income (PCPI) growth. The independent variables are initial PCPI, population density, distance to ports, and natural resource intensity.

- (a) Find the OLS coefficient estimates,  $b$ .
- (b) Provide a conditional graph of income growth on resource intensity. Verify that best fitting regression line for the bivariate “conditional” data produces the resource curse coefficient from part (a).

- (c) Use the matrix  $M^0$  to generate the goodness-of-fit measure,  $R^2$ .
  - (d) Perform a  $t$  test for the natural resource curse.
5. Gauss-Markov Theorem. Pick an alternative unbiased linear estimator to OLS. Show that if the Classical assumptions hold, OLS has a “tighter” sampling distribution.