# ECON 5350 Problem Set #3

Due: Monday, October 16 by 11:59 pm

**PAPER AND PENCIL PROBLEMS**

1. Prove that the matrix formula for the least squares estimator, $b = (X'X)^{-1}(X'Y)$, is equivalent to

$$\begin{aligned} b_1 &= \bar{y} - \bar{x}_2 b_2 \\ b_2 &= \frac{\sum_{i=1}^{n}(x_{i,2} - \bar{x}_2)(y_i - \bar{y})}{\sum_{i=1}^{n}(x_{i,2} - \bar{x}_2)^2} \end{aligned}$$

   when $k = 2$.

2. Prove that the matrix formula for the least squares estimator, $b = (X'X)^{-1}(X'Y)$, is equivalent to

$$\begin{aligned} b_1 &= \bar{y} - \bar{x}_2 b_2 - \bar{x}_3 b_3 \\ b_2 &= \frac{\sum_{i=1}^{n}(x_{i,2} - \bar{x}_2)(y_i - \bar{y})}{\sum_{i=1}^{n}(x_{i,2} - \bar{x}_2)^2} \\ b_3 &= \frac{\sum_{i=1}^{n}(x_{i,3} - \bar{x}_3)(y_i - \bar{y})}{\sum_{i=1}^{n}(x_{i,3} - \bar{x}_3)^2} \end{aligned}$$

   when $k = 3$ and $corr(x_2, x_3) = 0$.

3. Prove that the matrix formula for the variance of $b$, $var(b) = \sigma^2(X'X)^{-1}$, produces:

$$\begin{aligned} std.err.(b_2) &= \frac{\sigma}{\sqrt{\sum_{i=1}^{n}(x_{i,2} - \bar{x}_2)^2}} \\ std.err.(b_3) &= \frac{\sigma}{\sqrt{\sum_{i=1}^{n}(x_{i,3} - \bar{x}_3)^2}} \end{aligned}$$

   when $k = 3$ and $corr(x_2, x_3) = 0$.

**MATLAB PROBLEMS**

4. Use the state data in the file "resourcecurse.txt" and the regression model, $Y = X\beta + \epsilon$, to address the questions below. The sample period for economic growth is 1939-2004. The dependent variable is per capita personal income (PCPI) growth. The independent variables are initial PCPI, population density, distance to ports, and natural resource intensity.

   (a) Find the OLS coefficient estimates, $b$.

   (b) Provide a conditional graph of income growth on resource intensity. Verify that best fitting regression line for the bivariate "conditional" data produces the resource curse coefficient from part (a).

(c) Use the matrix $M^0$ to generate the goodness-of-fit measure, $R^2$.

(d) Perform a $t$ test for the natural resource curse.

5. Gauss-Markov Theorem. Pick an alternative unbiased linear estimator to OLS. Show that if the Classical assumptions hold, OLS has a "tighter" sampling distribution.