ECON 5350 Problem Set #6

Due: Tuesday, November 12, 2018

1. Simulate data ($n = 100$) for the model $y = \alpha x^{\beta} + \epsilon$ and parameter values $\alpha = 2$ and $\beta = 3$. Then use the simulated data, the Gauss-Newton algorithm, and NLS to estimate α and β. Repeat this procedure many, many times and show the sampling distributions of $\hat{\beta}$ for each estimation procedure. Comment on the results.

2. Consider the Poisson regression model

$$\Pr(Y_i = y_i|x_i) = \frac{e^{-\lambda_i}\lambda_i^{y_i}}{y_i!}$$

where $\lambda_i = \exp(\beta' x_i)$ and the log likelihood function is

$$\ln(L) = \sum_{i=1}^{n} \ln(\Pr(Y_i = y_i|x_i)).$$

   (a) Insert the expression for λ_i to obtain the log-likelihood function in terms of the observed data.

   (b) Derive the first-order conditions for maximizing this function with respect to β.

   (c) Derive the second derivatives matrix of this criterion function with respect to β.

   (d) Write out the full set of steps in an algorithm for obtaining the estimates of the parameters of this model. Include in your algorithm a test for convergence of the estimates based on Belsley's suggested criterion.

   (e) The following data were generated by the Poisson regression model with $\ln(\lambda) = \alpha + \beta x$

y	6	7	4	10	10	6	4	7	2	3	6	5	3	3	4
x	1.5	1.8	1.8	2.0	1.3	1.6	1.2	1.9	1.8	1.0	1.4	0.5	0.8	1.1	0.7

   Use your results from parts (a) through (d) to compute the maximum likelihood estimates of α and β.