

ECON 5350 Midterm Exam – Fall 2021

1. (50 pts) Consider the following (“true”) population regression model without a constant (i.e., $\beta_1 = 0$):

$$y_i = \beta_2 x_i + \epsilon_i,$$

where $i = 1, \dots, n$ and $\epsilon \sim i.i.d.N(0, \sigma^2)$.

- (a) (10 pts) Derive the OLS estimator for β_2 in summation form. Check the second-order condition.
 - (b) (10 pts) Write down the regression model in matrix form, carefully defining all variables. Show the equivalence of the matrix-based OLS estimator to the estimator in part (a).
 - (c) (10 pts) Consider a sample of data $X = (2, 3, 3, 0)'$ and $Y = (7, 6, 4, 3)'$. Calculate the R^2 value and comment on the results.
 - (d) (10 pts) Calculate the slope estimate with an intercept.
 - (e) (10 pts) The slope estimate in part (d) is different than the estimate in part (a). Does that imply one is biased? Explain.
2. (50 pts) Consider the beta(2,2) probability density function (pdf):

$$f(x) = Ax(1 - x),$$

where $0 \leq x \leq 1$.

- (a) (10 pts) Find the value of the constant A that ensures $f(x)$ is a valid pdf.
- (b) (10 pts) Find the cdf for X and verify the properties.
- (c) (10 pts) Find the mean and the variance of X , as well as the distribution of $Y = \ln X$.
- (d) (10 pts) Propose an unbiased estimator for the mean of X , μ_x . Do you know the small- and large-sample distributions for the estimator? If yes, what is the distribution?
- (e) (10 pts) Graph the distribution for X and describe a real-world random variable that could be described by the beta(2,2) distribution.