ECON 5350 Midterm Exam – Fall 2012

1. **Probability and Statistics (50 pts)**. Let X have a Pareto cdf where $F(x;\theta) = 1-(1/x)^{\theta}$ for $x \geq 1$ and zero elsewhere; $\theta > 3$.

   (a) Find the pdf for X, $f(x)$, and verify it is a valid pdf.

   (b) Find the mean and variance of X.

   (c) Let $\theta = 4$. Find the pdf for $Y = X^2$, $g(y)$. Find the mean of Y and verify that $g(y)$ is a valid pdf.

   (d) Now assume θ is unknown. Using the sample mean, $\bar{X}_n$, and population mean, $E(X)$, outline a procedure to test the null hypothesis H_0: $\theta = 4$ against H_A: $\theta \neq 4$. Assume you are given an i.i.d. random sample with a large n.

   (e) Consider two different values for the alternative in part (d), $\theta = 5$ and $\theta = 6$. Which will lead to a more powerful test and why? What is the power of the test in the region near the null. Explain.

2. **Classical Linear Regression Model (50 pts)**. Consider the following model: $Y_i = \beta_1 + \beta_2 X_i + \epsilon_i$ for $i = 1, ..., n$.

   (a) Without using matrices, derive the least squares estimator for the intercept, β_1.

   (b) Without using matrices, derive the least squares estimator for the slope, β_2.

   (c) Show that b_1 and b_2 are unbiased, make sure to highlight only the necessary Classical assumptions as you go.

   (d) Consider the hypothesis H_0: $\beta_1 = \beta_2$. Describe how you could test the null using a standard t test.

   (e) Now consider the alternative model $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \epsilon_i$, where $\bar{X}_2 = \bar{X}_3 = 0$ and $corr(X_{2i}, X_{3i}) = 0$. Use matrix algebra to find the least squares estimates of β_1, β_2 and β_3.