ECON 5350 Midterm Exam – Fall 2013

1. (**50 pts**) Consider the following ("true") population regression model without a constant (i.e., $\beta_1 = 0$):

$$y_i = \beta_2 x_i + \epsilon_i,$$

where $i = 1, ..., n$ and $\epsilon \sim i.i.d.N(0, \sigma^2)$.

   (a) (10 pts) Derive the OLS estimator for β_2 in summation form. Check the second-order condition.

   (b) (10 pts) Write down the regression model in matrix form, carefully defining all variables. Show the equivalence of the matrix-based OLS estimator to the estimator in part (a).

   (c) (10 pts) Consider a sample of data $X = (2, 3, 3, 0)'$ and $Y = (7, 6, 4, 3)'$. Calculate the R^2 value and comment on the results.

   (d) (10 pts) Calculate the slope estimate and R^2 with an intercept.

   (e) (10 pts) The slope estimate in part (d) is different than the estimate in part (a). Does that imply one is biased? Explain.

2. (**50 pts**) Consider the beta(2,2) probability density function (pdf):

$$f(x) = Ax(1 - x),$$

where $0 \leq x \leq 1$.

   (a) (10 pts) Find the value of the constant A that ensures $f(x)$ is a valid pdf.

   (b) (10 pts) Find the cdf for X and verify the properties.

   (c) (10 pts) Find the mean and the variance of X.

   (d) (10 pts) Find the distribution of $Y = \ln X$.

   (e) (10 pts) Graph the distribution for X and describe a real-world random variable that could be described by the beta(2,2) distribution.