

# ECON 5350 Final Exam – Fall 2013

Consider two competing production function models:

$$Q_t = \beta_1 K_t^{\beta_2} L_t^{\beta_3} + \epsilon_t \quad (1)$$

$$Q_t = \alpha_1 K_t^{\alpha_2} L_t^{\alpha_3} \exp(\epsilon_t) \quad (2)$$

where  $\epsilon_t \sim i.i.d. N(0, \sigma^2)$  and  $t = 1, \dots, T$ .

1. (25 pts) **Estimation.** Describe the least squares estimators for  $\alpha = (\alpha_1, \alpha_2, \alpha_3)'$  and  $\beta = (\beta_1, \beta_2, \beta_3)'$ .
2. (25 pts) **Hypothesis Testing.** Describe how to perform the tests of  $\beta_1 = 1$ ,  $\alpha_2 + \alpha_3 = 1$ , and  $\alpha_2 \alpha_3 = 1$ .
3. (25 pts) **Dummy Variables.** Using model #2, describe how to test the hypothesis that the quantity produced is higher on average after period  $t_*$ . In addition, describe how to test that the marginal productivity of labor is higher after period  $t_*$ .
4. (25 pts) **Maximum Likelihood Estimation.** The normal pdf for  $\epsilon_t$  is given by

$$f(\epsilon_t; \sigma^2) = (2\pi\sigma^2)^{-0.5} \exp(-0.5\epsilon_t^2\sigma^{-2}).$$

Calculate the ML estimators for  $\alpha = (\alpha_1, \alpha_2, \alpha_3)'$  and  $\beta = (\beta_1, \beta_2, \beta_3)'$ .