

## ECON 5350 Midterm Exam – Fall 2014

1. (50 pts) Consider the probability density function (pdf):

$$f(x) = \alpha x^\gamma, \text{ when } 0 \leq x \leq 1 \quad (1)$$

and zero elsewhere. Assume  $\gamma = 1$  for parts (a) through (d).

- (a) (10 pts) Find the cdf for  $X$  and verify the properties.
  - (b) (10 pts) Find the mean and the variance of  $X$ .
  - (c) (10 pts) Find the moment generating function (MGF) for  $X$ . Use the MGF to verify the mean and variance. Hint: The IBF formula is  $\int u dv = uv - \int v du$ .
  - (d) (10 pts) Find the distribution of  $Y = X^2$ . Verify that it is a valid pdf.
  - (e) (10 pts) Find the functional relationship  $\alpha = \alpha(\gamma)$  that ensures  $f(x)$  is a valid pdf. Assume  $\gamma \geq 0$ .
2. (50 pts) Consider the following population regression model:

$$y_i = \beta_1 + \beta_2 x_i + \epsilon_i, \quad (2)$$

where  $i = 1, \dots, n$ . Assume  $\epsilon \sim i.i.d.$  with the pdf in equation (1) and  $\gamma = 1$ .

- (a) (10 pts) Write down the regression model in matrix form, carefully defining all variables. Derive the matrix-based OLS estimator.
- (b) (10 pts) Which classical assumption is violated? What are the consequences?
- (c) (10 pts) Consider the sample  $X = (3, 1, 2, 2)'$  and  $Y = (3, 0, 1, 4)'$ . Calculate the intercept and slope estimates. What can you infer about the true intercept from your estimate?
- (d) (10 pts) Using the attached table, perform a  $t$  test that  $\beta_2 = 1$ . Discuss the validity of the test.
- (e) (10 pts) Find the joint pdf for the error terms. Substitute in equation (2) and take natural logs. The resulting function has a common name. What is it?