

ECON 5350 Midterm Exam – Fall 2015

1. **Probability and Statistics (60 pts).** Let X have the following pdf $f(x) = \theta x(1-x)$ for $0 \leq x \leq 1$ and zero elsewhere.

- (a) Find the value of θ that ensures that $f(x)$ is a valid pdf. Find the mean and variance of X .
- (b) Find the cdf for X . What is the probability that $0 \leq X \leq 0.5$?
- (c) Find the pdf for $Z = X^2$, $g(z)$. Is $g(z)$ a valid pdf? Verify.
- (d) Find the cdf for Z . What is the probability that $0 \leq Z \leq 0.25$? How does this compare to the answer in part (b)?
- (e) Let X_1, \dots, X_n be a random sample from $f(x)$ above. What is the mean and variance of n times the sample mean, $n\bar{X}_n$. What are the limiting and asymptotic distributions of \bar{X}_n ?
- (f) Derive the distribution of the n^{th} order statistic, Y_n , from a uniform $[0, 1]$ distribution. Find the mean of Y_n .

2. **Multiple Linear Regression and Least Squares (40 pts).**

- (a) Write out the multiple linear regression model in matrix form, carefully defining each matrix and its dimension. Then derive the least squares estimator in matrix form.
- (b) Derive the R^2 measure in matrix form. Now assume that the intercept is suppressed. Under what conditions will $R^2 < 0$?
- (c) Derive the least squares estimator (extensive form) for a bivariate regression model without an intercept.
- (d) The Gauss-Markov theorem states that the OLS estimator has the minimum variance among all linear, unbiased estimators provided the Classical assumptions hold. Go through the each of the six Classical assumptions and state one place where the assumption is used in proving the theorem. (NOTE: It is not necessary to formally complete the proof.)