

## ECON 5350 Midterm Exam – Fall 2017

1. **Probability and Statistics (50 pts).** Let  $X$  have a Pareto cdf where  $F(x; \theta) = 1 - (1/x)^\theta$  for  $x \geq 1$  and zero elsewhere;  $\theta > 3$ .
  - (a) Find the pdf for  $X$ ,  $f(x)$ , and verify it is a valid pdf.
  - (b) Find the mean and variance of  $X$ .
  - (c) Let  $\theta = 4$ . Find the pdf for  $Y = X^2$ ,  $g(y)$ . Find the mean of  $Y$  and verify that  $g(y)$  is a valid pdf.
  - (d) Outline two different procedures for estimating  $\theta$  from a random sample  $\{X_1, X_2, \dots, X_n\}$ .
  - (e) Find the pdf for the smallest value from a random sample of size  $n = 2$ ,  $\{X_1, X_2\}$ .
  
2. **Classical Linear Regression Model (50 pts).** Consider the following model:  $Y_i = \beta_1 + \beta_2 X_i + \epsilon_i$  for  $i = 1, \dots, n$ .
  - (a) Without using matrices, derive the least squares estimator for the intercept,  $\beta_1$ .
  - (b) Without using matrices, derive the least squares estimator for the slope,  $\beta_2$ .
  - (c) Show that  $b_1$  and  $b_2$  are unbiased, make sure to highlight only the necessary Classical assumptions as you go.
  - (d) Now consider the alternative model  $Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \epsilon_i$ , where  $\bar{X}_2 = \bar{X}_3 = 0$  and  $\text{corr}(X_{2i}, X_{3i}) = 0$ . Use matrix algebra to find the least squares estimates of  $\beta_1$ ,  $\beta_2$  and  $\beta_3$ .
  - (e) Assume the model in part (d) is the true population regression model, but you mistakenly estimate the following model:  $Y_i = \beta_1 + \beta_2 X_{2i} + \epsilon_i$ . Is the OLS estimate of  $\beta_2$  biased or unbiased? Defend your answer.