ECON 5350 Final Exam – Fall 2021

(200 pts.) Each question has equal weight.

1. **Linear Regression and Hypothesis Testing**. Consider the following regression model

$$y_i = \beta_1 + \beta_2 x_{2,i} + \beta_3 x_{3,i} + \epsilon_i$$

where $\epsilon_i \sim i.i.d.(0, \sigma^2)$, $e'e = 200$,

$$X'X = \begin{bmatrix} 103 & 0 & 0 \\ 0 & 200 & 0 \\ 0 & 0 & 50 \end{bmatrix} \text{ and } X'y = \begin{bmatrix} 206 \\ 300 \\ 50 \end{bmatrix}.$$

The formulae for the F statistics are

$$F = \frac{(Rb - q)'[R(X'X)^{-1}R']^{-1}(Rb - q)}{s^2 J} \text{ and } F = \frac{(e_*'e_* - e'e)}{s^2 J}.$$

(a) Find the least squares estimators for β_1, β_2 and β_3.

(b) Test the significance of β_2 and β_3. Use $t_{critical} = 2$ for the critical value.

(c) Test the overall goodness of fit. Use $F_{critical} = 3$ for the critical value.

(d) Find the least squares estimators for the restricted model with $\beta_3 = 0$.

(e) Perform an F test for the hypothesis H_o: $\beta_2 = \beta_3$.

2. **Nonlinear Regression Analysis**. Consider the following nonlinear (Box-Cox) regression model:

$$y_i = \beta(\frac{x_i^\lambda - 1}{\lambda}) + \epsilon_i$$

where ϵ_i, $i = 1, ..., N$ is a mean-zero, i.i.d normal ($f(\epsilon) = (2\pi\sigma^2)^{-n/2}\exp[-\epsilon'\epsilon/2\sigma^2]$) error term.

(a) Describe the procedure to obtain Gauss-Newton estimates of β and λ.

(b) Describe how to use the scanning/grid method to find estimates of β and λ.

(c) What is the elasticity of y_i with respect to x_i? How would you find the variance of the elasticity estimate?

(d) Find the maximum likelihood (i.e., maximum joint pdf) first-order condition for β.

(e) Describe how to use Newton's method to compute the ML estimates of β and λ.

3. **Generalized Regression Model**. Consider the following cross-sectional regression model:

$$y_i = \beta_1 + \beta_2 x_i + \epsilon_i$$

where $i = 1, ..., N$. The error terms are independent and mean-zero with variance:

$$\sigma_i^2 = \sigma^2 \exp(\gamma x_i).$$

Use the information above to answer the following five questions:

(a) Write the model in matrix form, clearly defining all matrices including the variance-covariance matrix for ϵ, Ω.

(b) Define a transformation matrix, P, that can be used to find efficient estimates of $\beta = (\beta_1 \ \beta_2)'$.

(c) Derive the efficient estimator and the associated standard errors.

(d) Provide some intuition for the weighting scheme in part (b).

(e) Discuss the small-sample and large-sample efficiency gains of the feasible GLS estimator as compared to OLS. When is it advisable to use OLS for prediction and inference? Explain your answers.