ECON 5350 Final Exam – Fall 2018

Consider two alternative regression models:

$$y_i = x_i^\beta + \epsilon_i \qquad \text{(Model \#1)}$$

and

$$y_i = \gamma x_i + \mu_i, \qquad \text{(Model \#2)}$$

where $i = 1, ..., n$, $\epsilon_i \sim i.i.d.\ N(0, \sigma_\epsilon^2)$, and $\mu_i \sim i.i.d.\ N(0, \sigma_\mu^2)$. You are given a dataset with three observations: $Y = (2, 4, 5)'$ and $X = (1, 2, 3)'$. Also, you may need to use $\frac{dA^x}{dx} = A^x \ln(A)$.

1. (25 pts) Derive the OLS estimator for γ, $\hat{\gamma}_{OLS}$. Then substitute in the data to get a point estimate.
2. (25 pts) Write down Model #2 in matrix form and show the equivalence of the matrix version and summation version of the OLS estimator.
3. (25 pts) Derive the variance of $\hat{\gamma}_{OLS}$. Then substitute in the data to get the standard error.
4. (25 pts) Set up the hypothesis test for $\gamma = 1$ including all the necessary information with a graph of the test statistic distribution. Discuss the size and power of your test, including any tradeoffs between the two.
5. (25 pts) Now assume $\mu_i = \mu_i^* x_i^2$, where $\mu_i^* \sim i.i.d.\ f(0, \sigma_\mu^2)$. Find the $var(\mu_i)$ and derive the efficient estimator for γ. What is the point estimate?
6. (25 pts) Perform one iteration of Gauss-Newton to estimate β.
7. (25 pts) Perform one iteration of nonlinear least squares using steepest descent to estimate β.
8. (25 pts) Describe in detail how would you would test $\frac{dy}{dx} = 1$ in model #1.