

ECON 5350 Final Exam – Fall 2017

Consider two alternative regression models without an intercept:

$$y_i = \beta \ln(x_i/z_i) + \epsilon_i \quad (\text{Model \#1})$$

and

$$y_i = \alpha_1 \ln(x_i) - \alpha_2 \ln(z_i) + \mu_i, \quad (\text{Model \#2})$$

where $i = 1, \dots, n$, $\epsilon_i \sim i.i.d. N(0, \sigma_\epsilon^2)$, and $\mu_i \sim i.i.d. N(0, \sigma_\mu^2)$.

1. (25 pts) Assume Model #2 is the correct model, but Model #1 is estimated. Derive the least-squares estimator of β in summation form. Show it is equivalent to the matrix-based estimator. Is the estimate unbiased?
2. (25 pts) Assume Model #1 is the correct model, but Model #2 is estimated. Derive the least-squares estimator of α_1 and α_2 in summation form. Are the estimates unbiased?
3. (25 pts) Assume that $s_{12} \neq 0$. What impacts will collinearity between x_i and z_i have on the properties of your least squares estimators of α_1 , α_2 and β ? Are the estimates efficient?
4. (25 pts) Under what condition will the OLS estimates satisfy $\hat{\alpha}_1 = \hat{\alpha}_2$? If this restriction holds, will it give the same estimate as $\hat{\beta}$?
5. (25 pts) Outline three equivalent procedures for testing the null hypothesis $H_0: \alpha_1 = \alpha_2$.
6. (25 pts) Describe in detail how to test the hypothesis that $\alpha_1\alpha_2 = 1$ or use the Box-Cox regression to distinguish between model #1 and the model $y_i = \beta(x_i/z_i) + \epsilon_i$. [Note: If your topic was Box-Cox regressions or the delta method, then you must choose the other one.]
7. (25 pts) Assume that $\sigma_{\epsilon_i}^2 = \sigma_\epsilon^2 [\ln(x_i/z_i)]^2$. Describe the efficient least squares estimator and contrast it with OLS. If you choose to report OLS estimates, what standard errors should you use?
8. (25 pts) Finally, assume the i.i.d. errors, ϵ_i , instead have pdf $f(\epsilon_i) = c \cdot (1 - \epsilon_i^2)$ in the range $[-1,1]$ and zero elsewhere. Sketch the pdf and find the constant c . Form the joint pdf of the errors, take a logarithmic transformation of the joint pdf, and then maximize the function with respect to β . What is the resulting estimate of β ? If it doesn't have a closed-form solution, then outline an estimation strategy.