ECON 5350 Midterm Exam – Fall 2016

1. **Probability and Statistics (50 pts)**. Let X have a Lomax pdf where $f(x;\alpha,\lambda) = (\alpha/\lambda)[1 + (x/\lambda)]^{-(\alpha+1)}$ for $x \geq 0$ and zero elsewhere; $\alpha, \lambda > 0$.

   (a) Verify that f is a valid pdf.

   (b) Find the cdf for X.

   (c) Find the mean of X when $\alpha = 2$ and $\lambda = 1$. [Hint: $\int \frac{x^m}{(1+x)^{n+1}} dx = \int \frac{x^{m-1}}{(1+x)^n} dx - \int \frac{x^{m-1}}{(1+x)^{n+1}}.$]

   (d) (BONUS 5 pts.) Find the variance of X when $\alpha = 2$ and $\lambda = 1$.

   (e) Find the distribution of $Y = X^2$.

   (f) Now assume α and λ are unknown. Using a random sample $\{X_i\}_{i=1}^n$, outline a procedure to estimate α and λ. Are $\hat{\alpha}$ and $\hat{\lambda}$ random or fixed? Explain.

2. **Classical Linear Regression Model (50 pts)**. Consider the following model: $Y_i = \beta_1 + \beta_2 X_i + \epsilon_i$ for $i = 1, ..., n$.

   (a) Write the model in matrix form and derive the least squares estimator, b.

   (b) Using the extensive form, show that b_2 is unbiased. Make sure to highlight all the necessary assumptions as you go.

   (c) Show that b is consistent, making sure to highlight all the necessary assumptions as you go.

   (d) Assume our class is your sample. Y is your midterm score; X is your GRE percentile score. Unfortunately, there is rampant cheating on the first midterm. Eyes are wandering, which creates a correlation between your score and that of your neighbor. Derive the variance-covariance matrix for b.

   (e) Assume the scores on the first exam are $Y = (85, 95, 65, 70, 80, 80, 75, 80, 100, 100)'$ with $X = (95, 90, 75, 70, 95, 80, 80, 80, 90, 95)'$. Find b_2 and the R^2. Comment on the results.