DELTA METHOD

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CASEY

Our distribution looks like:

$$\chi \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Starting with the Taylor Series approximation Theorem

$$g(x) \approx g(\mu) + g'(\mu)(x - \mu)$$

Take the expectation of both sides

$$E[g(x)] = E[g(\mu) + g'(\mu)(x - \mu)]$$

μ is a constant so we can pull out $g(\mu)$.

$$= g(\mu) + g'(\mu)E[(x - \mu)]$$

Expand the term inside the expectation

$$\begin{aligned} &= g(\mu) + g'(\mu)E[(x)] - E[(\mu)] \\ &= g(\mu) + g'(\mu)(\mu - \mu] \\ &= g(\mu) \end{aligned}$$

We know the expected value for $g(x)$ is $g(\mu)$, which gives the mean and now we need to find the variance.

$$\begin{aligned} Var(g(x)) &= E[g(x)^2] - E[g(x)]^2 \\ &= E[(g(\mu) + g'(\mu)(x - \mu))^2] - g(\mu)^2 \end{aligned}$$

Expand the expectation.

$$= E[g(\mu)^2 + 2g(\mu)(g'(\mu)(x - \mu)) + (g'(\mu)(x - \mu))^2] - g(\mu)^2$$

Pull out any constants from the expectation.

$$= g(\mu)^2 + 2g(\mu)g'(\mu)E[x - \mu] + E[(g'(\mu)(x - \mu))^2] - g(\mu)^2$$

Cancel out terms. (Remember $E[x - \mu] = 0$).

$$= E[(g'(\mu)(x - \mu))^2]$$

Expand.

$$= E[g'(\mu)^2(x - \mu)^2]$$

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Pull out constants.

$$= g'(\mu)^2 E[(x-\mu)^2]$$
$$= g'(\mu)^2 \frac{\sigma^2}{n}$$

By Slutsky's Theorem we can say the new distribution is

$$g(x) \xrightarrow{\text{D}} N\left(g(\mu), \frac{g'(\mu)^2\sigma^2}{n}\right)$$

Second-order Delta Method is very similar but is used when $g'(\mu) = 0$. Our distribution looks like:

$$\chi \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

Starting with the Taylor Series approximation Theorem

$$g(x) \approx g(\mu) + g'(\mu)(x-\mu) + \frac{g''(\mu)}{2}(x-\mu)^2$$
$$g(x) = g(\mu) + \chi^2 \frac{g''(\mu)}{2}$$
$$g(x) \xrightarrow{\text{D}} \chi^2\left(g(\mu), \frac{g''(\mu)^2\sigma^2}{2n}\right)$$