

ECON 5350 Solutions to the Midterm Exam

Spring 2008

Panel Data (100 pts). Consider the following panel data model

$$y_{i,t} = \alpha_i + x'_{i,t}\beta + \epsilon_{i,t}$$

where $i = 1, \dots, n$, $t = 1, \dots, T$ and one of the following four assumptions hold:

1. α_i is an unknown parameter and $\epsilon_{i,t} \sim i.i.d.(0, \sigma_\epsilon^2)$.
2. α_i is an unknown parameter and $\epsilon_{i,t} = \rho_i \epsilon_{i,t-1} + \nu_{i,t}$, where $\nu_{i,t} \sim i.i.d.(0, \sigma_\nu^2)$.
3. α_i is a random variable with mean α and $\epsilon_{i,t} \sim i.i.d.(0, \sigma_\epsilon^2)$.
4. α_i is a random variable with mean α , freely correlated across i , and $\epsilon_{i,t} \sim i.i.d.(0, \sigma_\epsilon^2)$.

For each of the four cases above, write out the full variance-covariance matrix of the errors and outline an estimation strategy that will produce consistent and asymptotically efficient estimates of β .

Solutions.

Variance-Covariance Matrices

Case #1. $\Omega_1 = \sigma_\epsilon^2 I_{nT}$

$$\text{Case \#2. } \Omega_2 = \begin{bmatrix} \Sigma(1)_T & 0 & \cdots & 0 \\ 0 & \Sigma(2)_T & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma(n)_T \end{bmatrix}$$

where

$$\Sigma(i)_T = \frac{\sigma_\nu^2}{(1 - \rho_i^2)} \begin{bmatrix} 1 & \rho_i & \cdots & \rho_i^{T-2} & \rho_i^{T-1} \\ \rho_i & 1 & \cdots & \rho_i^{T-3} & \rho_i^{T-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \rho_i^{T-2} & \rho_i^{T-3} & \cdots & 1 & \rho_i \\ \rho_i^{T-1} & \rho_i^{T-2} & \cdots & \rho_i & 1 \end{bmatrix}.$$

Case #3. $\Omega_3 = I_n \otimes \Sigma_T$

where

$$\Sigma_T = \begin{bmatrix} \sigma_\epsilon^2 + \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \sigma_\alpha^2 & \sigma_\epsilon^2 + \sigma_\alpha^2 & \cdots & \sigma_\alpha^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_\alpha^2 & \sigma_\alpha^2 & \cdots & \sigma_\epsilon^2 + \sigma_\alpha^2 \end{bmatrix}.$$

$$\text{Case \#4. } \Omega_4 = \Omega_3 + [\Sigma_n \otimes i_T i_T']$$

where

$$\Sigma_n = \begin{bmatrix} 0 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & 0 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & 0 \end{bmatrix}$$

is a symmetric matrix and i_T is a $(T \times 1)$ column vector of ones.

Estimation Strategies

The estimation procedures below will all produce consistent and asymptotically efficient estimates.

1. This is the fixed effects (FE) model, so it can be estimated using OLS with a series of cross-sectional dummies capturing the fixed effects.
2. This is a FE model with serial correlation, so it can be estimated using GLS and a series of cross-sectional dummies. The two-step estimator will first estimate ρ_i using the OLS residuals:

$$\hat{\rho}_i = \frac{\sum_{t=2}^T (e_{i,t} - \bar{e}_i)(e_{i,t-1} - \bar{e}_i)}{\sum_{t=1}^T (e_{i,t} - \bar{e}_i)^2}$$

and then apply the feasible GLS formula

$$\hat{\beta} = (X' \hat{\Omega}_2^{-1} X)^{-1} (X' \hat{\Omega}_2^{-1} Y).$$

3. This is the random effects (RE) model, so it can be estimated using GLS. The procedure is outlined in my lecture notes and in chapter 13 of Greene's text.
4. This is a variant of the RE model, so it can also be estimated using GLS. Everything is the same as in my lecture notes except for the fact that the $\sigma_{ij}, i \neq j$ need to be estimated. One method for estimating these is to use

$$\hat{\sigma}_{ij} = \sum_{t=1}^T (e_{i,t} - \bar{e}_i)(e_{j,t} - \bar{e}_j).$$