ECON 5360 Solutions to the Midterm Exam

Spring 2009

Panel Data (100 pts). Consider the following two-way panel data model

$$y_{i,t} = \alpha_i + x'_{i,t}\beta + \gamma_t + \epsilon_{i,t}$$

where $i = 1, ..., n$, $t = 1, ..., T$ and one of the following four assumptions hold:

1. α_i and γ_t are unknown parameters and $\epsilon_{i,t}$ is a mean-zero independent random variable with variance $\sigma^2_{\epsilon,i}$.
2. α_i and γ_t are unknown parameters and $\epsilon_{i,t} = \rho\epsilon_{i,t-1} + \nu_{i,t}$, where $\nu_{i,t} \sim i.i.d.(0, \sigma^2_\nu)$.
3. γ_t is an unknown parameter, α_i is a mean-zero independent random variable with variance $\sigma^2_{\alpha,i}$, and $\epsilon_{i,t} \sim i.i.d.(0, \sigma^2_\epsilon)$.
4. α_i is an unknown parameter, γ_t is a mean-zero random variable, $\gamma_t = \rho\gamma_{t-1} + \nu_t$, where $\nu_t \sim i.i.d.(0, \sigma^2_\nu)$ and $\epsilon_{i,t} \sim i.i.d.(0, \sigma^2_\epsilon)$.

Random variables α, γ and ϵ are mutually independent. For each of the four cases above, write out the full variance-covariance matrix of the errors when $n = 2$ and $T = 3$. For cases 1 and 2, outline an estimation strategy that will produce consistent and asymptotically efficient estimates of β when n and T are large.

Solutions.

Variance-Covariance Matrices

$$\text{Case \#1. } \Omega_1 = \Sigma \otimes I_T = \begin{bmatrix} \sigma^2_{\epsilon,1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{\epsilon,1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{\epsilon,1} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{\epsilon,2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{\epsilon,2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{\epsilon,2} \end{bmatrix}$$

where

$$\Sigma = \begin{bmatrix} \sigma^2_{\epsilon,1} & 0 \\ 0 & \sigma^2_{\epsilon,2} \end{bmatrix}.$$

$$\text{Case \#2. } \Omega_2 = \Sigma \otimes \Phi_T = \frac{\sigma_\nu^2}{1-\rho^2} \begin{bmatrix} 1 & \rho & \rho^2 & 0 & 0 & 0 \\ \rho & 1 & \rho & 0 & 0 & 0 \\ \rho^2 & \rho & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho & \rho^2 \\ 0 & 0 & 0 & \rho & 1 & \rho \\ 0 & 0 & 0 & \rho^2 & \rho & 1 \end{bmatrix}$$

where

$$\Sigma = \begin{bmatrix} \frac{\sigma_\nu^2}{1-\rho^2} & 0 \\ 0 & \frac{\sigma_\nu^2}{1-\rho^2} \end{bmatrix} \text{ and } \Phi_T = \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix}.$$

$$\text{Case \#3. } \Omega_3 = \begin{bmatrix} \sigma_\epsilon^2 + \sigma_{\alpha,1}^2 & \sigma_{\alpha,1}^2 & \sigma_{\alpha,1}^2 & 0 & 0 & 0 \\ \sigma_{\alpha,1}^2 & \sigma_\epsilon^2 + \sigma_{\alpha,1}^2 & \sigma_{\alpha,1}^2 & 0 & 0 & 0 \\ \sigma_{\alpha,1}^2 & \sigma_{\alpha,1}^2 & \sigma_\epsilon^2 + \sigma_{\alpha,1}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_\epsilon^2 + \sigma_{\alpha,2}^2 & \sigma_{\alpha,2}^2 & \sigma_{\alpha,2}^2 \\ 0 & 0 & 0 & \sigma_{\alpha,2}^2 & \sigma_\epsilon^2 + \sigma_{\alpha,2}^2 & \sigma_{\alpha,2}^2 \\ 0 & 0 & 0 & \sigma_{\alpha,2}^2 & \sigma_{\alpha,2}^2 & \sigma_\epsilon^2 + \sigma_{\alpha,2}^2 \end{bmatrix}.$$

$$\text{Case\#4. } \Omega_4 = \begin{bmatrix} \sigma_\epsilon^2 + \sigma_\gamma^2 & \rho\sigma_\gamma^2 & \rho^2\sigma_\gamma^2 & \sigma_\gamma^2 & \rho\sigma_\gamma^2 & \rho^2\sigma_\gamma^2 \\ \rho\sigma_\gamma^2 & \sigma_\epsilon^2 + \sigma_\gamma^2 & \rho\sigma_\gamma^2 & \rho\sigma_\gamma^2 & \sigma_\gamma^2 & \rho\sigma_\gamma^2 \\ \rho^2\sigma_\gamma^2 & \rho\sigma_\gamma^2 & \sigma_\epsilon^2 + \sigma_\gamma^2 & \rho^2\sigma_\gamma^2 & \rho\sigma_\gamma^2 & \sigma_\gamma^2 \\ \sigma_\gamma^2 & \rho\sigma_\gamma^2 & \rho^2\sigma_\gamma^2 & \sigma_\epsilon^2 + \sigma_\gamma^2 & \rho\sigma_\gamma^2 & \rho^2\sigma_\gamma^2 \\ \rho\sigma_\gamma^2 & \sigma_\gamma^2 & \rho\sigma_\gamma^2 & \rho\sigma_\gamma^2 & \sigma_\epsilon^2 + \sigma_\gamma^2 & \rho\sigma_\gamma^2 \\ \rho^2\sigma_\gamma^2 & \rho\sigma_\gamma^2 & \sigma_\gamma^2 & \rho^2\sigma_\gamma^2 & \rho\sigma_\gamma^2 & \sigma_\epsilon^2 + \sigma_\gamma^2 \end{bmatrix}$$

where $\sigma_\gamma^2 = \sigma_\nu^2/(1-\rho^2)$.

Estimation Strategies

The estimation procedures below will produce consistent and asymptotically efficient estimates.

1. This is the two-way fixed effects (FE) model with groupwise heteroscedasticity. It can be estimated using GLS with a series of cross-sectional and time-series dummies capturing both types of fixed effects. The two-step estimator will first estimate

$$\hat{\sigma}_{\epsilon,i}^2 = (e_i'e_i)/T,$$

where e_i, $i = 1, ..., n$, is a vector of the OLS residuals for the i^{th} cross section. Using these estimates, we then apply the feasible GLS estimator:

$$\hat{\beta} = (X'\hat{\Omega}_1^{-1}X)^{-1}(X'\hat{\Omega}_1^{-1}Y).$$

2. This is the two-way fixed effects (FE) model with common serial correlation. It can be estimated using GLS with a series of cross-sectional and time-series dummies capturing both types of fixed effects. The two-step estimator will first estimate ρ:

$$\hat{\rho} = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{\sum_{t=2}^{T} e_{i,t}e_{i,t-1}}{\sum_{t=1}^{T} e_{i,t}^2}\right]$$

and then use $\hat{\rho}$ to apply the feasible GLS estimator:

$$\hat{\beta} = (X'\hat{\Omega}_2^{-1}X)^{-1}(X'\hat{\Omega}_2^{-1}Y).$$