ECON 5350 Midterm Exam – Spring 2007

1. **Generalized Least Squares (25 pts)**. Consider the following partitioned regression model:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \beta + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \tag{1}$$

where

$$\Omega = \begin{bmatrix} \sigma_1^2 I_{n_1} & 0 \\ 0 & \sigma_2^2 I_{n_2} \end{bmatrix}$$

is the variance covariance matrix of the error terms. Find the formula for the GLS estimator and suggest a feasible GLS estimator.

2. **Autocorrelation (30 pts)**. Consider the simple linear regression model $y_t = \beta_0 + \beta_1 x_t + \epsilon_t$, where the errors exhibit second-order autocorrelation: $\epsilon_t = \rho_2 \epsilon_{t-2} + \mu_t$.

   (a) Calculate the autocovariance function (i.e., $\gamma(s) = cov(\epsilon_t, \epsilon_{t-s})$) under the assumption that ϵ_t is a weakly covariance stationary process.

   (b) Assuming ρ_2 is known, derive the GLS estimator of β_1.

   (c) Describe how one would calculate an asymptotically efficient two-step estimator of β_1 if ρ_2 were unknown.

3. **SUR and Panel-Data Models (45 pts)**. Consider the following SUR system:

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} X_1 & 0 \\ 0 & X_2 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \tag{2}$$

where Y_1, Y_2, ϵ_1 and ϵ_2 are of dimension $(T \times 1)$; X_j is of dimension $(T \times k_j)$ for $j = 1, 2$; β_j is of dimension $(k_j \times 1)$ for $j = 1, 2$; and the system variance-covariance matrix is

$$\Omega = \Sigma \otimes I_T = \begin{bmatrix} \sigma_1^2 I_T & \sigma_{12} I_T \\ \sigma_{12} I_T & \sigma_2^2 I_T \end{bmatrix}$$

where

$$\Omega^{-1} = \Sigma^{-1} \otimes I_T = \begin{bmatrix} a_{11} I_T & a_{12} I_T \\ a_{12} I_T & a_{22} I_T \end{bmatrix}.$$

(a) The SUR system above could be used to incorporate panel data. Describe an estimation strategy to obtain consistent, asymptotically efficient estimates for a fixed effects version of Model (2).

(b) Show that equation-by-equation OLS is equivalent to GLS when $X_1 = X_2$. For simplicity, consider the case when $k_1 = k_2 = 1$ and $T = 2$.

(c) Describe, in detail, how to test Model (1) versus Model (2) when $\sigma_{12} = 0$. Continue to assume that $X_1 = X_2$, $k_1 = k_2 = 1$ and $T = 2$.