

# **Pollution Permits, Green Taxes, and the Environmental Poverty Trap**

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July 9, 2018

## **Abstract**

We compare pollution permits and green taxes in a unified overlapping generations model with endogenous longevity. The model identifies pollution permits as a potential source of multiple equilibria. One nontrivial equilibrium is an environmental poverty trap (EPT) with low capital and a high stock of pollution. An economy operating around the equilibrium will gravitate toward this equilibrium in the long run. The other nontrivial equilibrium is a desirable one with high capital and a low stock of pollution. A saddle path leads to this desirable equilibrium. Alternatively, green taxes produce a unique stable equilibrium that avoids the EPT. Our conclusion is that developing countries can continue to consider pollution permits as an efficient mechanism to improve environmental conditions but proceed with caution given the possibility of being drawn into an EPT.

***JEL Classification:*** C61; O44; Q58

***Keywords:*** Pollution Permits; Green Taxes; Saddle Path

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## **1. Introduction**

Developing countries face pressing economic and environmental challenges. Economic growth in developing countries usually comes with severe environmental degradation in terms of worsening air, water, and soil quality. According to a report by the World Bank Group, the readings of air pollution indicators are higher in low-and-middle-income countries than in high-income countries (World Bank Group, 2015a). The health effect of pollution is staggering. Environmental degradation is estimated to be responsible for about 40% of world deaths (Pimentel et al., 2007). Air pollution alone is estimated to cause 3.3 million premature deaths each year around the world, notably in China and India (Lelieveld, Evans, Fnais, Giannadaki, & Pozzer, 2015). To address environmental deterioration, market-based environmental regulations, pollution permits and green taxes, are available for developing countries. These two regulations have appealed to economists because they provide the correct incentive for polluters to efficiently decide pollution levels (Stavins, 1998; Hanley, Shogren, & White, 2007). Both policies, targeting a wide spectrum of pollutants, have been well established in developed countries and achieved some success, such as in the United States, the European Union, Switzerland, New Zealand, Japan, Canada, and South Korea (Levinson, 2007; Borisova & Roka, 2010; EPA, 2015; European Commission, 2015; ICAP, 2018). Developing countries, however, are just in their initial stages of implementing pollution permits and green taxes. Kazakhstan in 2013 initiated a carbon emission trading system, and at the end of 2017, China launched the world's largest carbon market. Further, Ukraine, Mexico, Turkey, Chile, Columbia, Brazil, Thailand, and Vietnam plan to implement an emissions trading scheme. Green taxes also have been implemented in developing countries such as Mexico, Chile, Columbia, and China (ICAP, 2018; Xinhua, 2018).

But can the market-based environmental regulations make the situation worse in developing countries if they are not implemented properly? In this paper, we address that question by comparing the dynamics of the economy and the environment in a developing country that is considering either pollution permits or green taxes to manage the environment. Our analysis shows that pollution permits, if not properly managed, may endogenously give rise to an environmental poverty trap (EPT) whereas green taxes do not. An EPT refers to an equilibrium outcome with low capital and high pollution, and emerges in dynamic economic and environmental systems with multiple equilibria.<sup>1</sup> The idea of EPT is not new, but this paper identifies a new mechanism that contributes to the emergence of EPT. EPTs have been shown to be generated by threshold effects of environmental quality on longevity or survival probability (Mariani, Pérez-Barahona, & Raffin, 2010), endogenous technical choice in the presence of emission taxes (Varvarigos, 2014), nonlinear regeneration of environmental quality (Prieur, Jean-Marie, & Tidball, 2013; Dao & Edenhofer, 2014), and public pollution abatement financed by public debt (Fodha & Seegmuller, 2014). The emergence of EPT in this paper, however, does not depend on the mechanisms identified by the extant literature, but instead depends only on usual assumptions in terms of technology, preferences, and the law of motion for the stock of pollution in the context of pollution permits. The primary contribution of this paper thus enriches the literature by providing a new mechanism that may potentially lead to an EPT.

The possibility of an EPT under pollution permits is revealed by the marriage of two modeling frameworks, Stokey (1998) and Chakraborty (2004). These two frameworks are used by a wide spectrum of theoretical literature that unifies the environment with economic growth.

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<sup>1</sup> In this paper, we define an EPT in the context of multiple equilibria. Although some developing countries, such as China and India, currently have relatively bad environmental quality and low levels of GDP per capita, it does not necessarily follow that these countries are stuck in EPTs. To conclude whether a country is stuck in an EPT, one needs to calibrate the model to resemble that country and see whether the model exhibits multiple equilibria.

Pollution is modeled as associated with either production or consumption. In line with this modeling approach, the Stokey (1998) framework allows for mathematical transformation of pollution as being a byproduct of final output into a necessary input in production. By tailoring the interpretation of the price of pollution in production within the Stokey (1998) framework, we flexibly analyze and compare the dynamic systems under pollution permits and under green taxes in a unified modeling framework. Pollution nevertheless comes with consequences on economic growth. The most common modeling approach is that pollution imposes detrimental effect on agents' utility or on productivity (see Xepapadeas, 2005 for a fine summary). Pollution also can affect economic growth by endogenously altering agents' time preference (Chu, Lai, & Liao, 2015; Vella, Dioikitopoulos, & Kalyvitis, 2015). In recent years, the literature has specifically shed light on the effects of pollution on economic growth through the health channel. Because pollution makes people sick, the effective labor supply is reduced (Chen, Shieh, & Chang, 2015). Pollution increases morbidity, so agents engage in precautionary savings to pay for medical expenses (Wang, Zhao, & Bhattacharya, 2015). Pollution also affects mortality/longevity/life expectancy, thus modifying agents' incentive to save (Pautrel, 2009; Jouvet, Pestieau, & Ponthiere, 2010; Varvarigos, 2010; Raffin & Seegmuller, 2014; Varvarigos, 2014; Raffin & Seegmuller, 2017). The common feature of the last strand of literature is that it augments the OLG model with endogenous longevity developed by Chakraborty (2004). In this strand of literature, longevity is affected negatively by pollution, while positively by factors such as private and public healthcare spending as well as income per capita. By linking longevity, pollution, savings, and capital within the Chakraborty (2004) framework, we establish the connection between the economy and the environment. To the best of our knowledge, this paper

is the first to combine the frameworks of Stokey (1998) and Chakraborty (2004) to compare pollution permits and green taxes.

We also note that this paper is related to the seminal article by Weitzman (1974), who compares economic control via prices or via quantities in a static world. Weitzman argues that it is equivalent to regulate an autarky economy by restricting either prices or quantities under certainty, but it may be not equivalent under uncertainty. We instead consider a dynamic world and find the same equivalence in a certain world, but also highlight a key difference between environmental regulations with prices versus with quantities. Although the same steady-state level of welfare can be achieved with either a properly selected green-tax rate or a pollution quota, the choice of control can have important implications for the existence of multiple equilibria and the associated transition dynamics. Sim and Lin (2018) also point out the difference between quantity control versus price control in a certain world, but they focus on a globalized and static economy rather than on an autarky and dynamic setting used in this paper. Therefore, another contribution of this paper is to fully consider the transition dynamics, thus augmenting the ‘prices vs. quantities’ debate surrounding the appropriate nature of environmental regulation.

Inspired by the literature on endogenous longevity, we consider a basic two-period OLG model with agents’ longevity improved by income per capita but reduced by the stock of pollution. The stock of pollution accumulates due to economic activity and declines due to public environmental maintenance. If some technical conditions are satisfied, multiple equilibria endogenously arise under pollution permits. One nontrivial equilibrium is an EPT with low capital and high pollution. The economy will be trapped in the EPT unless the government intervenes to steer the economy towards the other nontrivial equilibrium, a desirable one with

high capital and low pollution. Alternatively, no EPT exists under green taxes because there is only one stable equilibrium. We also consider an alternative model that further incorporates private healthcare expenditure as another endogenous factor improving agents' longevity. The main results qualitatively carry through.

The existence of multiple equilibria under pollution permits can be explained as follows. The law of motion for capital implies a negative relationship between pollution and capital. A high stock of pollution implies a low level of capital since agents save little due to their short longevity. Similarly, a low stock of pollution implies a high level of capital due to high savings. Can these two combinations be supported in the long run under environmental regulation? With pollution permits, the flow of pollution is fixed but a high level of capital implies more funds for environmental maintenance, leading to a low stock of pollution. Conversely, a low level of capital leads to insufficient funds for environmental maintenance so that the stock of pollution is high. Either combination can be supported in the long run. Under green taxes, however, emissions are not fixed but are relatively low when the economy is depressed. This implies a low level of capital and a low stock of pollution. As the economy grows and the level of capital becomes higher, emissions are higher and the stock of pollution is larger. This means that the only feasible combination of pollution and capital under green taxes lies somewhere between the two extremes, and thus multiple equilibria are not supported.

We wish to highlight three main results. First, a developing economy implementing pollution permits might be pulled into an EPT unless the government encourages agents to increase savings. In contrast, a developing economy using green taxes will always converge to the unique equilibrium that can replicate the desirable equilibrium under pollution permits, such that the EPT can be avoided. Second, a well-designed environmental regulation with pollution

permits where the government intervenes in savings can generate approximately the same discounted sum of welfare as under green taxes along the transition path. Third, multiple equilibria under pollution permits are more likely to emerge for pollutants that are relatively easy to clean up. The main policy implication of our research is that developing countries can continue to consider pollution permits as an efficient mechanism to improve environmental conditions but proceed with caution given the possibility of being drawn into an EPT.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 describes the equilibria and dynamics when pollution permits are implemented. Section 4 describes the equilibrium and dynamics when green taxes are employed. Section 5 provides numerical simulations to (i) contrast the key variables along the transition paths, (ii) compare the long-run impacts from changes in parameters under pollution permits versus under green taxes, and (iii) investigate how likely the emergence of multiple equilibria is under pollution permits. Section 6 discusses practical implications of the model and concludes.

## 2. The Basic Setup

### 2.1 The production function

The production function is specified as

$$Y_t = BK_t^\alpha L_t^\beta P_t^{1-\alpha-\beta}, \quad (1)$$

where  $K_t$  is physical capital,  $L_t$  is the aggregate labor supply,  $P_t$  is the flow of pollution,

$B \in (0, +\infty)$  is a production function scalar, and  $\alpha, \beta \in (0, 1)$  are parameters with  $1 - \alpha - \beta > 0$ .

This production function is in line with the common modeling method where the flow of pollution

is a byproduct of final output. After some mathematical manipulation of the Stokey (1998) model, the flow of pollution can equivalently be treated as a necessary input in the production function (Ono, 2002; Brock & Taylor, 2005; Jouvét, Michel, & Rotillon, 2005b, 2005a; Prieur et al., 2013). This is a convenient modeling framework for comparing pollution permits and green taxes.

## 2.2 Firms

The economy is perfectly competitive. In each period, the representative firm faces a static profit-maximization problem subject to the production function (1). Denote  $r_t$  as the rental price of capital,  $w_t$  as wage rate, and  $q_t$  as the “price” of pollution, all of which are exogenous to the firm. The term  $q_t$  can be interpreted either as the price of pollution permits if the government auctions pollution permits to the firm<sup>2</sup> or as the marginal tax rate if the government levies a green tax on the firm. Although  $q_t$  can have two different interpretations, the profit-maximization problem for the firm takes the same form. The firm chooses capital  $K_t$ , labor  $L_t$ , and the flow of pollution  $P_t$  as production inputs to maximize profits  $\pi_t$ . The problem is

$$\max_{K_t, L_t, P_t} \pi_t = BK_t^\alpha L_t^\beta P_t^{1-\alpha-\beta} - r_t K_t - w_t L_t - q_t P_t.$$

The first-order conditions require the marginal product of each input to equal its price:

$$r_t = \alpha B k_t^{\alpha-1} p_t^{1-\alpha-\beta}, \quad (2)$$

$$w_t = \beta B k_t^\alpha p_t^{1-\alpha-\beta}, \quad (3)$$

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<sup>2</sup> Pollution permits can be distributed to firms either by “grandfathering”, i.e., the government directly gives pollution permits to firms for free, or by auctioning. Jouvét et al. (2005b) show that all pollution permits should be auctioned, because auctioning is the most efficient way of distributing pollution permits. Although the model can be easily adapted to allow for the case of “grandfathering”, we focus on the case where pollution permits are auctioned.

$$q_t = (1 - \alpha - \beta) B k_t^\alpha p_t^{-\alpha - \beta}, \quad (4)$$

where  $k_t = K_t/L_t$  and  $p_t = P_t/L_t$  are capital and flow of pollution in per-capita terms. Perfect competition drives the representative firm's profits to zero.

Consider the case in which the government auctions pollution permits to manage the environment. The government sets a pollution quota per capita,  $\bar{p}_t$ , and auctions off the pollution permits representing the quota. Equation (4) then determines each firm's willingness to pay for pollution permits with  $q^{pp}(k_t, \bar{p}_t)$  equal to the equilibrium auction price of pollution permits.<sup>3</sup> The representative firm treats  $\bar{p}_t$  as exogenous and  $q^{pp}(k_t, \bar{p}_t)$  is set endogenously through the auctioning of pollution permits. The equilibrium price of pollution permits increases in capital and decreases in the amount of pollution allowed by the permits.

Also consider the case in which the government levies green taxes to manage the environment. In this case,  $q_t$  is interpreted as the tax rate per unit of pollution emitted rather than as the auction price of pollution permits. With green taxes, the firm treats tax rate  $q_t$  as an exogenous variable set by the government and the firm chooses the optimal level of emissions,  $p^{gt}(k_t, q_t)$ , determined by (4). The flow of pollution emitted increases in capital and decreases in the green-tax rate.

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<sup>3</sup> In equilibrium, the amount of pollution that firms choose as a production input,  $p_t$ , is equal to the amount of pollution allowed by pollution permits,  $\bar{p}_t$ . We assume that the government consistently cuts pollution, implying that  $\bar{p}_t$  is always a binding constraint.

## 2.3 Government

The government maintains a balanced budget in each period. We assume that each period's fiscal revenues from controlling pollution,  $q_t P_t$ , serve as the sole source of income for the government.

With these fiscal revenues, the government finances public environmental maintenance  $A_t$  that prevents pollution from growing.<sup>4</sup> Denote  $a_t = A_t/L_t$  as per-capita public environmental maintenance. If the government implements environmental regulation with pollution permits, the balanced budget in per-capita terms is

$$a_t^{pp} = q^{pp}(k_t, \bar{p}_t) \bar{p}_t = (1 - \alpha - \beta) B k_t^\alpha \bar{p}_t^{1-\alpha-\beta}. \quad (5)$$

Alternatively, if the government uses green taxes, the balanced budget in per-capita terms is

$$a_t^{gt} = q_t p^{gt}(k_t, q_t) = (1 - \alpha - \beta) \left( \frac{1-\alpha-\beta}{q_t} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}. \quad (6)$$

## 2.4 Pollution

The stock of pollution  $Z_t$  accumulates due to the flow of pollution  $P_t$  and declines due to environmental maintenance  $A_t$ . The stock of pollution evolves according to

$$Z_{t+1} = (1 - \theta) Z_t - \gamma A_t + P_t, \quad (7)$$

where  $\theta \in (0, 1]$  measures the speed of autonomous dissipation of the stock of pollution, and

$\gamma > 0$  is a parameter representing the efficiency of environmental maintenance. Dividing

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<sup>4</sup> Examples of environmental maintenance include building water treatment plants to reduce waste water, cloud seeding, deploying water cannons, and erecting giant air cleaners (Nature, 2018) to reduce urban PM2.5 and PM10 density, as well as planting trees to reduce atmospheric carbon dioxide level.

equation (7) through by  $L_t$  and assuming no population growth yields the evolution of the per-capita stock of pollution

$$z_{t+1} = (1 - \theta)z_t - \gamma a_t + p_t. \quad (8)$$

## 2.5 Agents

A representative agent lives two periods, which is divided into “young” and “elderly” categories. Following Chakraborty (2004), an agent lives the entirety of her young period but only lives the fraction  $\phi$  of her elderly period.<sup>5</sup> An agent born at the beginning of period  $t$  therefore has lifetime longevity equal to  $1 + \phi_{t+1}$ . Assume that  $\phi_{t+1}$  depends on the agent’s health status in her young period,  $x_t$ . An agent’s health status improves in income per capita but deteriorates due to the stock of pollution. Similar to Varvarigos (2014) and supported by empirical findings such as Ebenstein et al. (2015), we treat the health status  $x_t$  as the ratio of income per capita to the adjusted stock of pollution, i.e.,  $x_t = y_t / \eta z_t$ , where  $y_t = Y_t / L_t$  and  $\eta > 0$  captures the detrimental effect of pollution on the health status.<sup>6</sup> Assume

$$\phi_{t+1} = \phi(x_t) = \frac{\lambda + \bar{\lambda} x_t}{1 + x_t}, \quad (9)$$

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<sup>5</sup> The term  $\phi$  can be interpreted as either the fraction of the elderly period that an agent lives or the survival probability at the beginning of the elderly period. Interpreting  $\phi$  as survival probability does not affect the predictions of the model, but it can raise bequest issues. Because all agents save in the young period, but a fraction of the agents will die at the beginning of the elderly period, the savings of those who die will become income for those who are alive in the elderly period. Bequeathing thus leads the return rate of savings to be larger than the rental rate of capital. In contrast, if  $\phi$  is interpreted as deterministic longevity, the return rate of savings will be equal to the rental rate of capital.

<sup>6</sup> In Appendix D, we consider an alternative model to check the robustness of the results. In this alternative model, the agent’s health status is determined by private healthcare expenditure, income per capita, and the stock of pollution. Although agents still treat income per capita and the stock of pollution as given, they can actively improve their longevity through private healthcare expenditure. We show that the main results derived from the basic model remain robust to this alternative specification.

where the parameters  $\underline{\lambda}$  and  $\bar{\lambda}$  are the lower and upper bounds of a representative agent's potential longevity in the elderly period, i.e.,  $0 \leq \underline{\lambda} \leq \phi_{t+1} < \bar{\lambda} \leq 1$ . Substituting  $x_t = y_t / \eta z_t$  into (9) gives an agent's longevity as a function of income per capita  $y_t$  and the stock of pollution  $z_t$ :

$$\phi_{t+1} = \phi(y_t, z_t) = \frac{\underline{\lambda} \eta z_t + \bar{\lambda} y_t}{\eta z_t + y_t}. \quad (10)$$

Note that the representative agent takes both income per capita and the stock of pollution as given.

The representative agent born at the beginning of period  $t$  derives utility from consumption in her youth,  $c_t$ , and elderly consumption,  $d_{t+1}$ . Longevity is taken as given by the agent, but all else equal, she will derive more utility from consumption if she lives longer in her elderly period. To permit closed-form solutions, we assume that the representative agent's lifetime utility takes a logarithmic form that is additively separable (see, for example, Chakraborty, 2004; Fodha & Seegmuller, 2014; Varvarigos, 2014):

$$U_t = \ln c_t + \phi_{t+1} \ln d_{t+1}. \quad (11)$$

When the representative agent is young, she inelastically supplies one unit of labor to earn the exogenous wage rate  $w_t$ . The agent's labor income is divided between young consumption,  $c_t$ , and savings,  $s_t$ . Savings can only be rented to firms in the form of physical capital at the rental price of  $r_{t+1}$ . The remunerated savings,  $r_{t+1}s_t$ , are returned to the agent at the beginning of her elderly period and are used to finance her elderly consumption,  $d_{t+1}$ .

The representative agent faces two budget constraints – one for her young period and one for her elderly period:

$$w_t = c_t + s_t, \quad (12)$$

$$r_{t+1}s_t = d_{t+1}. \quad (13)$$

Solving the agent's utility-maximization problem gives the saving function

$$s_t = \Phi_{t+1} w_t = \frac{\phi_{t+1}}{\phi_{t+1} + 1} w_t, \quad (14)$$

where  $\Phi_{t+1} = \phi_{t+1}/(\phi_{t+1} + 1) \in \left[ \frac{\underline{\lambda}}{\underline{\lambda} + 1}, \frac{\bar{\lambda}}{\bar{\lambda} + 1} \right)$  is the agent's propensity to save. Because the agent's propensity to save increases in longevity, savings increase in longevity if other things are equal.

By (12) and (13), the agent's young and elderly optimal levels of consumption are

$$c_t = \frac{1}{\phi_{t+1} + 1} w_t, \quad (15)$$

$$d_{t+1} = r_{t+1} \frac{\phi_{t+1}}{\phi_{t+1} + 1} w_t. \quad (16)$$

Other things equal, equation (15) indicates that the longer the agent lives, the lower consumption will be in her youth as the agent has to save more to satisfy her elderly consumption. Conversely, equation (16) suggests that the longer the agent lives, all else equal, the higher her elderly consumption will be.

### 3. Environmental Regulation with Pollution Permits

We now investigate the dynamics of the economy and the environment under environmental regulation with pollution permits. For simplicity, we assume that physical capital completely depreciates within one period, and hereafter treat physical capital as a flow rather than as a stock. Since agents' savings in period  $t$  become physical capital in period  $t+1$ , we have

$$k_{t+1} = s_t. \quad (17)$$

Substituting equation (14) into (17) gives

$$k_{t+1} = \Phi(y_t, z_t)w_t, \quad (18)$$

where  $\Phi_{t+1} = \Phi(y_t, z_t)$  is the agent's propensity to save as a function of income per capita and the stock of pollution. Consider constant per-capita pollution permits,  $\bar{p}_t = \bar{p}$  for all  $t$ .

Substituting expressions into (18) under environmental regulation with pollution permits yields the nonlinear difference equation for physical capital:

$$k_{t+1} = \frac{\underline{\lambda}\eta z_t + \bar{\lambda}B\bar{p}^{1-\alpha-\beta}k_t^\alpha}{(\underline{\lambda}+1)\eta z_t + (\bar{\lambda}+1)B\bar{p}^{1-\alpha-\beta}k_t^\alpha} \beta B\bar{p}^{1-\alpha-\beta}k_t^\alpha. \quad (19)$$

From equation (19), we define  $k_{t+1} - k_t = 0$  as the  $kk^{pp}$  locus under pollution permits:

$$\text{the } kk^{pp} \text{ locus: } \frac{\underline{\lambda}\eta z_t + \bar{\lambda}B\bar{p}^{1-\alpha-\beta}k_t^\alpha}{(\underline{\lambda}+1)\eta z_t + (\bar{\lambda}+1)B\bar{p}^{1-\alpha-\beta}k_t^\alpha} \beta B\bar{p}^{1-\alpha-\beta}k_t^\alpha - k_t = 0. \quad (20)$$

Similarly, we have a difference equation for the stock of pollution

$$z_{t+1} = (1-\theta)z_t - \gamma a^{pp}(k_t, \bar{p}) + \bar{p}, \quad (21)$$

which after substitutions gives

$$z_{t+1} = (1 - \theta)z_t - \gamma(1 - \alpha - \beta)B\bar{p}^{1-\alpha-\beta}k_t^\alpha + \bar{p}. \quad (22)$$

Finally, from equation (22), we define  $z_{t+1} - z_t = 0$  as the  $zz^{pp}$  locus under pollution permits:

$$\text{the } zz^{pp} \text{ locus: } -\theta z_t - \gamma(1 - \alpha - \beta)B\bar{p}^{1-\alpha-\beta}k_t^\alpha + \bar{p} = 0. \quad (23)$$

For  $k_0, z_0 > 0$ , the dynamics of the economy and the environment under environmental regulation with pollution permits are determined jointly by difference equations (19) and (22).

In the subsequent analysis, we make use of the following

*Assumptions: (i)  $0 < \alpha < \beta < 1$ , and (ii)  $\underline{\lambda} = 0$  and  $\bar{\lambda} = 1$ .*

Assumption (i) says capital's share in production is smaller than labor's share, which is consistent with the empirical evidence (see, for example, Mankiw, Romer, & Weil, 1992; Gollin, 2002). This assumption, combined with the condition  $1 - \alpha - \beta > 0$ , implies that  $0 < \alpha < 1/2$ .

Assumption (ii) will greatly simplify the math but does not qualitatively alter the results.

### 3.1 Steady States

We summarize the conditions for the emergence of multiple equilibria in Proposition 1. Then we focus on the interesting case where multiple equilibria emerge.

*Proposition 1. When  $\gamma \leq \frac{2}{1-\alpha-\beta} \frac{\theta}{\eta}$ , a unique equilibrium exists; when  $\gamma > \frac{2}{1-\alpha-\beta} \frac{\theta}{\eta}$ , and*

*$\frac{1}{\eta} \beta B^2 \bar{p}^{2(1-\alpha-\beta)} < T(k^*)$ , where  $T(k_t) = \frac{\bar{p}}{\theta} k_t^{1-2\alpha} + \left[ \frac{2}{\eta} - \frac{\gamma}{\theta} (1 - \alpha - \beta) \right] B \bar{p}^{1-\alpha-\beta} k_t^{1-\alpha}$  and*

$$k^* = \left[ \frac{1-2\alpha}{1-\alpha} \frac{\bar{p}^{\alpha+\beta}}{[\gamma(1-\alpha-\beta) - \frac{2\theta}{\eta}] B} \right]^{\frac{1}{\alpha}}, \text{ multiple (dual) equilibria emerge.}$$

*Proof.* See Appendix B, Proof #1.

Proposition 1 provides a quantitative guidance for policymakers to guard against multiple equilibria when a pollutant is regulated by pollution permits. The proposition says that for the environmental maintenance efficiency  $\gamma$ , there exists a threshold value  $\frac{2}{1-\alpha-\beta} \frac{\theta}{\eta}$ , which consists of a scalar  $\frac{2}{1-\alpha-\beta}$  determined by the upper bound of longevity and pollution's share in production, and the dissipation rate for the stock of pollution  $\theta$  adjusted by the detrimental effective of pollution on longevity  $\eta$ . If  $\gamma$  does not surpass this threshold value, the pollutant is relatively difficult to clean up and only one nontrivial steady state exists. An example is carbon dioxide. As a green-house gas, carbon dioxide is technically difficult to eliminate from the atmosphere and the value for  $\gamma$  is small. Since carbon dioxide does not impose severe health problems, the value for  $\eta$  is small, which leads to a relatively large value for the adjusted dissipation rate of pollution  $\theta/\eta$ . So it is likely that a unique equilibrium exists when carbon dioxide is regulated by pollution permits. However, if  $\gamma$  is larger than this threshold value, the pollutant is relatively easy to clean up. This fact, combined with an additional technical condition, gives rise to the emergence of multiple equilibria. A case in point is water pollutants. It is technically easier to clean up water pollutants and the value for  $\gamma$  is large. However, water pollutants are poisonous and the value for  $\eta$  is large. Thus, the adjusted dissipation rate  $\theta/\eta$  is relatively small. So multiple equilibria may emerge when water pollutants are managed by pollution permits.

Figure 1 illustrates the capital-pollution dynamics with multiple equilibria under pollution permits. The  $kk^{pp}$  locus defines all combinations of  $k_t$  and  $z_t$  where capital is in steady state. The  $kk^{pp}$  locus slopes down because, all else equal, a lower stock of pollution leads to increased longevity, more savings, and a higher level of capital. The  $zz^{pp}$  locus defines all combinations of

$k_t$  and  $z_t$  where the stock of pollution is in steady state. The  $zz^{pp}$  locus slopes down because economic growth drives up the demand for pollution permits. This causes the price of pollution permits to increase and raises public environmental maintenance, which in turn reduces the stock of pollution. The analytical proofs of the slopes of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus are shown in Appendix B. There are three steady states – two nontrivial steady states and  $k = z = 0$ .<sup>7</sup> The two nontrivial steady states are labeled  $(k^l, z^h)$  and  $(k^h, z^l)$ , where the superscripts denote “low” and “high”. The former steady state leads to reduced longevity, low young and elderly consumptions, low welfare, and thus should be avoided. In contrast, the latter steady state is associated with improved longevity, high young and elderly consumptions, high welfare, and thus is desirable.

What is the intuition behind the existence of two steady states? The key is that environmental maintenance increases in capital for a given value of  $\bar{p}$  (see equation (5)). When the level of capital is low, the environment deteriorates because the discrepancy is relatively large between the flow of pollution allowed by  $\bar{p}$  and the effective environmental maintenance. In the undesirable equilibrium, agents’ longevity is reduced due to the deteriorated environment and they save less, thus reinforcing the fact that the level of capital is low and the stock of pollution is high. When the level of capital is high, however, the environment improves because the discrepancy is relatively small between the flow of pollution allowed by  $\bar{p}$  and the effective environmental maintenance. In the desirable equilibrium, agents’ longevity is improved due to

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<sup>7</sup> Point A in Figure 1 is a trivial steady state where the economy shuts down. Capital declines to zero because the stock of pollution converges to positive infinity such that agents’ longevity is zero. Individuals do not save for consumption in the elderly period of life, and no capital is generated. Because capital is a necessary input in production, the economy shuts down and no pollution will be emitted. The human impact on the evolution of stock of pollution is gone and the stock of pollution gradually goes back to its natural equilibrium, i.e., the stock of pollution is zero.

the ameliorated environment and they save more, thus reinforcing the fact that the level of capital is high and the stock of pollution is low.

Consider the following example to highlight the intuition behind the EPT. Suppose a developing economy is poor, the government revenues can only support the building of a few water treatment plants. As there is a large gap between the waste water discharged allowed by the number of pollution permits and the total capacity of water treatment plants, the water quality will be poor, which in turn reduces agents' longevity and savings. Since savings are low, the economy will be depressed with a low level of capital, which will not generate enough resources to finance the building of any additional water treatment plants. The cycle continues and the country is stuck in an EPT. Alternatively, if the government can encourage sufficient savings, more water treatment plants can be built.<sup>8</sup> The gap between waste water discharged and the capacity of water treatment plants is small, so an ameliorated water quality will lead to increased longevity, savings, and welfare. The EPT is avoided, but it may require government intervention to push the economy onto the desirable transition path. We next turn to a more rigorous examination of the dynamic properties around the steady states under pollution permits.

### 3.2 Transition Dynamics

We summarize the transition dynamics under pollution permits in the following proposition:

*Proposition 2. The steady state  $(k^l, z^h)$  is a stable equilibrium. The steady state  $(k^h, z^l)$  is saddle if  $z^l > \bar{p} / \left( \frac{1+\alpha}{\alpha} \frac{2-\theta}{E^{pp}} + 2 \right)$  and is unstable if  $z^l < \bar{p} / \left( \frac{1+\alpha}{\alpha} \frac{2-\theta}{E^{pp}} + 2 \right)$ , where  $E^{pp}$  is the elasticity of the propensity to save with respect to income per capita evaluated at the steady state  $(k^h, z^l)$ .*

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<sup>8</sup> Although not explicitly modeled in this paper, the government can take advantage of the bully pulpit to encourage agents to adjust savings, or resort to fiscal incentives, such as a consumption tax and tax breaks in retirement savings.

*Proof.* See Appendix B, Proof #4.

We are interested in the case where the desirable steady state can be reached and exhibits saddle stability, and draw directional arrows around the steady states in Figure 1. The directional arrows show that point B is a locally stable EPT. For an initial stock of pollution in the vicinity of point B, there is a continuum of capital levels (savings choice in the previous period) that will cause the system to gravitate toward point B. This is an undesirable equilibrium path that results in a relatively low level of capital and a high stock of pollution. In contrast, the directional arrows around point C (i.e., the desirable steady state) show a saddle path with a unique level of capital (savings) that allows the economy to converge to the desirable steady state with a high level of capital and a low stock of pollution.

#### 4. Environmental Regulation with Green Taxes

We now consider the equilibrium properties of the economy and the environment in which green taxes are imposed to regulate the environment. Applying equation (18) with a constant tax rate  $q_t = q$  on the flow of pollution gives the difference equation for physical capital

$$k_{t+1} = \frac{\underline{\lambda}\eta z_t + \bar{\lambda} \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}}{(\underline{\lambda}+1)\eta z_t + (\bar{\lambda}+1) \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}} \beta \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}. \quad (24)$$

Using equation (24), we define  $k_{t+1} - k_t = 0$  as the  $kk^{st}$  locus under green taxes:

$$\text{the } kk^{st} \text{ locus: } \frac{\underline{\lambda}\eta z_t + \bar{\lambda} \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}}{(\underline{\lambda}+1)\eta z_t + (\bar{\lambda}+1) \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}} \beta \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} - k_t = 0. \quad (25)$$

The law of motion for the stock of pollution is

$$z_{t+1} = (1 - \theta)z_t + (1 - \gamma q) \left( \frac{1 - \alpha - \beta}{q} \right)^{\frac{1}{\alpha + \beta}} B^{\frac{1}{\alpha + \beta}} k_t^{\frac{\alpha}{\alpha + \beta}}. \quad (26)$$

From equation (26), we define  $z_{t+1} - z_t = 0$  as the  $zz^{gt}$  locus under green taxes:

$$\text{the } zz^{gt} \text{ locus: } -\theta z_t + (1 - \gamma q) \left( \frac{1 - \alpha - \beta}{q} \right)^{\frac{1}{\alpha + \beta}} B^{\frac{1}{\alpha + \beta}} k_t^{\frac{\alpha}{\alpha + \beta}} = 0. \quad (27)$$

For  $k_0, z_0 > 0$ , equations (24) and (26) jointly determine the capital-pollution dynamics in an economy with green taxes being implemented.

#### 4.1 Steady State

In this section, we examine the steady state for the case where green taxes are implemented.

Figure 2 shows both the  $kk^{gt}$  locus with all the combinations of  $k_t$  and  $z_t$  where capital is in steady state, and the  $zz^{gt}$  locus with all the combinations of  $k_t$  and  $z_t$  where the stock of pollution is in steady state. Similar to Figure 1, the  $kk^{gt}$  locus slopes down because less pollution leads to increased longevity, more savings, and a higher level of capital. However, the  $zz^{gt}$  locus slopes up. The  $zz^{gt}$  locus slopes up because unlike under pollution permits, the flow of pollution is not fixed under green taxes but continues to rise with economic growth. The fact that the  $kk^{gt}$  locus slopes down and the  $zz^{gt}$  locus slopes up implies that there is only one nontrivial steady state<sup>9</sup> and no EPT under a green-tax system. This is one of our primary findings. The proofs that the  $kk^{gt}$  locus slopes down and the  $zz^{gt}$  locus slopes up are shown in Appendix C.

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<sup>9</sup> In order to examine the non-trivial solution area, we focus on the case of interest where  $1 - \gamma q > 0$ . This case implies that the flow of pollution is greater than the effective environmental maintenance in each period, i.e.,

## 4.2 Transition Dynamics

We summarize the transition dynamics under green taxes in the following proposition:

*Proposition 3. If Assumption (i)  $0 < \alpha < \beta < 0$  holds, the system always converges to the unique steady state under green taxes. Let  $E^{gt}$  be the elasticity of propensity to save with respect to income per capita evaluated at the unique steady state under green taxes. When  $E^{gt}$  satisfies  $\left[ \frac{\alpha}{\alpha+\beta}(E^{gt} + 1) - (1 - \theta) \right]^2 - 4\theta \frac{\alpha}{\alpha+\beta} E^{gt} < 0$ , the eigenvalues are complex conjugates and the convergence is cyclical; when  $E^{gt}$  satisfies  $\left[ \frac{\alpha}{\alpha+\beta}(E^{gt} + 1) - (1 - \theta) \right]^2 - 4\theta \frac{\alpha}{\alpha+\beta} E^{gt} > 0$ , the eigenvalues are real and distinct and the convergence is non-cyclical.*

*Proof.* See Appendix C, Proof #2.

The directional arrows in Figure 2 show that the transition dynamics around the nontrivial steady state (point B) are characterized by cycles provided that the eigenvalues are complex conjugates. For a given initial stock of pollution, any choice for the level of capital (i.e., previous period's savings) will cause the system to cycle into point B. There are some implications associated with the dampened cycle toward the unique steady state under green taxes. For example, the dampened cycle gives rise to intergenerational inequality/inequity (Seegmuller & Verchère, 2004; Schumacher & Zou, 2015) and periodically negative correlation between longevity and economic growth (Varvarigos, 2013).

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$p^{gt}(k, q) - \gamma qp^{gt}(k, q) > 0$ . First, the case makes intuitive sense in that a pollutant that is hard to eliminate (represented by a low value for  $\gamma$ ) justifies a high green-tax rate  $q$  to control the pollutant from its source. Second, the case is a common assumption in the literature. Economides and Philippopoulos (2008) argue that it is “too good to be true” that the effective cleanup activity is greater than the polluting effect of production activity. Raffin and Seegmuller (2014) argue that this assumption “enforces an additional upper bound only on the environmental tax rate, and consequently on the level of preventive expenses.” Third, this assumption is inherently consistent with the law of motion for the stock of pollution when the government implements pollution permits. By equation (21), the existence of a non-trivial solution area also implies that the flow of pollution is larger than the effective environmental maintenance when pollution permits are implemented.

## 5. Numerical Simulations: Pollution Permits versus Green Taxes

In this section, we provide numerical simulations to further illustrate the differences between a system regulated by pollution permits versus a system regulated by green taxes. First, we use a set of benchmark parameters to solve the social planner's problem (See Appendix A) and calculate the socially optimum value for  $\bar{p}$  at the steady state. Second, we compare the key variables along the transition paths. Third, in the context of multiple equilibria under pollution permits, we compare how changes in the parameters affect the steady states. Fourth, to complement the analysis of the long-run effects on steady states, we focus on the sensitivity of multiple equilibria under pollution permits to different combinations of parameters.

### 5.1 Comparison of the Transition Paths

Consider a developing economy with a degraded environment that is attempting to reach a desirable equilibrium with high capital and a low stock of pollution. If the government uses pollution permits, it needs to encourage agents in the initial period to adjust savings, thus affecting future capital and placing the economy on the saddle path. If the government resorts to green taxes, however, it only needs to design a proper green-tax rate on the flow of pollution and the system will converge to the desirable equilibrium. The open question is which environmental regulation – pollution permits or green taxes – generates higher welfare along the transition path. To make the two environmental regulations comparable, we start at a common initial point featuring lower capital and a higher stock of pollution relative to those determined by the desirable equilibrium. we also calibrate the green-tax rate within the model so that the unique equilibrium under green taxes is identical to the desirable equilibrium under pollution permits. We consider two scenarios depending on whether the saddle path is reached under pollution

permits – one where the government encourages agents to adjust savings to the level on the saddle path and a second scenario where there is no government intervention in savings. We apply numerical simulations to compare these two environmental regulations with the calibrated and calculated parameters shown in Table 1.

Consider the scenario in which the government implementing pollution permits encourages agents to increase savings in the initial period, such that the level of capital is on the saddle path leading to the desirable equilibrium.<sup>10</sup> To compare, we assume only the young generation lives in the initial period, and further define period welfare as the summation of welfare for the young and old agents living in the same period. We denote period 0 as the initial period, so the comparisons between elderly consumption, longevity, and period welfare begin in period 1. We plot the time paths of key variables under pollution permits versus under green taxes in Figure 3. The notable feature of Figure 3 is that while the starting point and the desirable equilibrium are identical across the two environmental regulations, the environmental regulation with pollution permits gives rise to larger fluctuations along the transition paths than the environmental regulation with green taxes. The oscillations under pollution permits are spurred by the large increase in savings. Because the government encourages agents to increase savings in period 0, there is a large increase in the level of capital from period 0 to period 1 and the young consumption in period 0 is greatly reduced relative to that under green taxes. The increase in savings allows for higher consumption during the elderly phase of the lifecycle and for the next generation of young individuals (period 1). However, the following generation of young agents (period 2) choose to optimally reduce their consumption, save more, and shift

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<sup>10</sup> It is possible that a developing economy may not have the national resources available for savings to ensure the economy can jump onto the saddle path in one period. In this case, the nation would need to increase savings over multiple periods, borrow from other nations, or rely on international aid. In the benchmark model and the subsequent simulations, the economy has sufficient wage income to reach the required level of savings in one period and jump onto the saddle path that leads to the desirable long-run equilibrium.

consumption to the elderly phase of the lifecycle. This occurs because longevity is expected to increase due to the sharp reduction in the stock of pollution from environmental maintenance supported by the jump in capital in period 1. After a sharp increase in welfare in period 3 due to the contribution of a greatly boosted longevity caused by the small stock of pollution in period 2, the back-and-forth movement in consumption and other variables eventually dampens as the economy transitions to the desirable equilibrium. In contrast, the green-tax system operates smoothly along the transition path because the flow of pollution is not fixed by pollution permits but determined by the level of capital, which limits the fluctuations in pollution-driven longevity. Despite the differences across environmental regulations, the discounted sum of welfare under pollution permits over the 20 periods along the transition path is approximately the same as that under green taxes.<sup>11</sup>

Figure 4 compares the transition paths under pollution permits versus under green taxes in the scenario where there is no government intervention in savings. While the economy regulated by pollution permits gravitates toward the EPT, the economy under green taxes cycles to the desirable equilibrium with a high level of capital and a low stock of pollution.

Given that environmental regulation with pollution permits generates larger fluctuations in the economy and has the potential to drag a developing economy into an EPT, green taxes might be a safer environmental management choice within our framework. Under green taxes, the economy will always converge to the unique equilibrium with small fluctuations in terms of key economic and environmental variables. This unique equilibrium can replicate the desirable equilibrium under pollution permits, such that the EPT can be avoided. We note, however, that a

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<sup>11</sup> The discounted sum of welfare under pollution permits is 3.64, whereas the discounted sum of welfare under green taxes is 3.44. The difference in discounted sum of welfare is equivalent to 12.14% of the welfare generated by the initial young consumption under pollution permits, or to 10.66% of the welfare generated by the initial young consumption under green taxes, indicating the difference is relatively modest.

carefully constructed environmental regulation with pollution permits that achieves saddle-path stability is capable of generating approximately the same discounted sum of social welfare along the transition path. To achieve this outcome with pollution permits, policymakers must encourage the current generation of young agents to sacrifice consumption and save more. This will ensure a high level of capital in the following period and place the economy on the saddle path leading to the desirable equilibrium.

Another policy concern is whether the EPT persists if the number of pollution permits is allowed to change over time. Policymakers may wish to gradually issue fewer permits to allow for an ever-improving environment. To explore this issue, suppose the government decreases the number of pollution permits from the benchmark value by 2% each period relative to the previous period. The effects of the decreases in pollution permits, all else equal, are organized in Figure 5. We can see that the EPT still exists even if policymakers continually decrease  $\bar{p}$ , and it is still necessary for policymakers to intervene in terms of encouraging young agents' savings.

## **5.2 Comparison of the Long-Run Impacts from Changes in the Parameters**

In this section, we compare the long-run effects of parameter changes under pollution permits versus under green taxes. We focus on parameters related to environmental regulation ( $\bar{p}$  under pollution permits and  $q$  under green taxes), environmental maintenance efficiency ( $\gamma$ ), the detrimental effect of pollution on the health status ( $\eta$ ), and the dissipation rate of the stock of pollution ( $\theta$ ). The environmental regulation parameters are set by the government, while the latter three depend on the type of pollutant and its subsequent health impact. In Appendix E, we provide analytical results of how the parameters affect the level of capital and the stock of pollution in the steady state under the two types of environmental regulations.

Under pollution permits, we specifically consider the case where multiple equilibria exist<sup>12</sup>, and the second column of Table 2 gives the ranges of each parameter that give rise to multiple equilibria while holding other parameters constant. The signs in Table 2 show how the level of capital, the stock of pollution, longevity, and period welfare in the steady state change as the key parameters increase under pollution permits. An increase in  $\bar{p}$  contributes to wage income and fiscal revenues at the cost of the environment. For the stable, undesirable steady state (i.e., the EPT), an increase in  $\bar{p}$  is to the detriment of both the environment and the economy. Due to the low level of capital, public environmental maintenance does not keep up with the increase in the flow of pollution. The environment worsens and agents' longevity is reduced. Agents respond by saving less, thus further increasing the stock of pollution because fewer resources are available for environmental maintenance. In contrast, an increase in  $\bar{p}$  improves the conditions of the environment and the economy in the desirable steady state. Because of the high level of capital, the increase in the flow of pollution is counterbalanced by even more environmental maintenance. The environment is ameliorated, and agents' longevity increases and save more. Our results echo the findings by Ono (2002) that decreases in pollution permits could lead to lower capital and environmental quality in the long run. The rationale of Ono (2002) is that there is a critical value for pollution permits at which the positive income effect is balanced by the negative one, such that both the long-run capital and environmental quality are maximized. The rationale behind our result is different and instead depends on the type of equilibrium. When multiple equilibria emerge, both steady states entail competing effects on the environment and thus on longevity through the flow of pollution versus public environmental

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<sup>12</sup> The equilibria must lie in the first quadrant, and must be either stable or saddle such that they can be reached.

maintenance. So how pollution permits alter the relative magnitudes of the competing effects varies depending on the type of equilibrium in which the economy is operating.

The effects of  $\gamma$ ,  $\eta$ , and  $\theta$  on the undesirable steady state are as expected, but the effects on the desirable steady state seem counter-intuitive. To understand the seemingly counter-intuitive results, consider the transition path to the steady state after an increase in environmental maintenance efficiency,  $\gamma$ . The entire  $zz^{pp}$  is shifted to the left and the downward sloping  $kk^{pp}$  locus gives rise to a new desirable steady state and a new saddle path to the left of the original ones. The new saddle path requires that individuals must be encouraged to save less, such that the level of capital in the next period is lower. The lower level of capital reduces environmental maintenance expenditure, so the steady-state stock of pollution increases in spite of the more efficient environmental maintenance. As the economy transitions to the new steady state, the level of capital is lower and the stock of pollution is higher, such that longevity and period welfare are reduced. Arguments for the other two parameters,  $\eta$  and  $\theta$ , are similar.

Table 3 summarizes how the level of capital, the stock of pollution, longevity, and period welfare in the steady state change with the fundamental parameters under green taxes. A higher value for the green-tax rate  $q$  (i.e., a more stringent environmental regulation) is conducive to both the economy and the environment in the long run. The intuition is that a higher value for  $q$  imposes two opposite effects on the steady-state level of capital. A higher  $q$  imposes a positive effect by enhancing agents' longevity through the reduced stock of pollution, but imposes a negative effect by reducing income per capita. With the benchmark parameters, the positive effect of a higher  $q$  outweighs the negative effect, and thus the steady-state level of capital increases in  $q$ . It follows that both steady-state longevity and period welfare increase. However,

a higher green-tax rate does come with a short-run cost along the transition path. In the period following a rise in the green-tax rate  $q$ , the level of capital and period welfare temporarily decrease. Therefore, an increase in the green-tax rate harms the economy in the short run but benefits the economy in the long run.

Although the signs associated with  $\gamma$  and  $\theta$  are as expected, the long-run effects of  $\eta$  seem counter-intuitive. An increase in  $\eta$  decreases both the level of capital and the stock of pollution in the steady state. The reason is that for any given stock of pollution, a higher value for  $\eta$  decreases the steady-state level of capital because agents' longevity is reduced by more harmful pollution and they save less. A lower level of capital in turn decreases the marginal product of pollution. Firms respond by polluting less until the marginal product of pollution matches the green-tax rate again. The negative effect of a lower level of capital outweighs the positive effect of less pollution, leading to decreases in both longevity and welfare.

The comparison of Table 2 and Table 3 reveal the following two results. First, for a more stringent environmental control (a smaller  $\bar{p}$  and a larger  $q$ ), the long-run impacts on the stable, undesirable steady state under pollution permits are the same as those on the unique steady state under green taxes. Second, under pollution permits, a less stringent environmental policy (i.e., a larger  $\bar{p}$ ) is welfare improving for the desirable steady state. The intuition for the last result is that developed economies are better off to pollute more, grow the economy, and invest in the necessary infrastructure to clean up the environment.

### **5.3 Sensitivity Analysis of Multiple Equilibria under Pollution Permits**

The second column of Table 2 shows the ranges of each parameter that lead to the emergence of multiple equilibria while holding other parameters constant. This section further investigates how

sensitive the emergence of multiple equilibria is under pollution permits to different combinations of policy and technical parameters.

Table 4 shows the combinations of the dissipation rate of the stock of pollution and the number of pollution permits ( $\theta$  on the horizontal axis and  $\bar{p}$  on the vertical axis of each graph) under different sets of technical parameters ( $\eta$  and  $\gamma$ ) that give rise to multiple equilibria under pollution permits. First, all else equal, as the value for  $\theta$  increases, the range of  $\bar{p}$  that gives rise to multiple equilibria first expands and then shrinks. And there exists a positive correlation between  $\theta$  and  $\bar{p}$  that can generate multiple equilibria. Second, a larger value for  $\gamma$  allows for a higher possibility of multiple equilibria. This observation is consistent with Proposition 1. Third, the emergence of multiple equilibria is not sensitive to the change in the value for  $\eta$ . The exercise in Table 4 directs our attention to the type of pollutant that may give rise to an EPT if it is regulated by pollution permits. Take carbon dioxide and water pollutants for example. Because carbon dioxide is associated with a low value for  $\gamma$  and a low value for  $\eta$ , carbon dioxide lies in the top-left corner of Table 4. In contrast, water pollutants have a high value for  $\gamma$  and a high value for  $\eta$ . So water pollutants lie in the bottom-right corner of Table 4 and are more likely to generate multiple equilibria under pollution permits. We thus conclude that multiple equilibria emerge under a wider range of parameters for some pollutants.

## 6. Conclusions and Discussion

In the basic two-period OLG model with standard assumptions regarding technology, preferences, and dynamics for the stock of pollution, we show that pollution permits may give rise to multiple equilibria whereas green taxes do not. Under pollution permits, one nontrivial equilibrium is an

EPT acting as a “sink”. The other nontrivial equilibrium is a desirable one exhibiting saddle stability. A developing economy implementing pollution permits might be pulled into an EPT unless the government encourages agents to increase savings to the level on the saddle path. In contrast, green taxes give rise to a unique stable equilibrium, such that an EPT can be avoided. A developing economy implementing green taxes will always converge to the unique equilibrium and the green-tax rate can be chosen to replicate the desirable equilibrium under pollution permits. Along the transition path, a well-designed environmental regulation with pollution permits can generate approximately the same discounted sum of welfare as under green taxes. Developing countries can continue to consider pollution permits as an efficient mechanism to improve environmental conditions. However, permits targeting a pollutant that is relatively easy to clean up may give rise to multiple equilibria, so the government should proceed with caution given the possibility of being drawn into an EPT. The main findings are robust to an alternative model where agents can actively engage in private healthcare efforts to prolong longevity.

In practice, the popularity of pollution permits is growing among developing countries. China, for example, has recently become an advocate and user of pollution permits to control its carbon emissions. It is likely that more developing countries will implement pollution permits to target a broader range of pollutants other than just carbon dioxide. We note that pollutants differ in their dissipation rates, in their threats posed on longevity, and in their difficulty to be cleaned up. These concepts are captured in our model by the select parameters, which allow us to relate our findings to the real world. Our findings are thus likely to have increasing future relevance and can serve as a guide for developing countries to avoid the pitfalls of being drawn into EPTs. The adoption of green taxes in developing countries is also on the rise. One recent example is that China on January 1, 2018 launched environmental protection taxes. Although no EPT

emerges under green taxes within the model, developing countries using green taxes still need to carefully choose the tax rate, such that the long-run outcome is comparable to the desirable equilibrium under pollution permits.

### **Acknowledgements**

This paper is supported by the Fundamental Research Funds for the Central Universities (No. 531107051097) and the project “Research on Formation Mechanism, Measurement System and Promotion Method of Environmental Resource Allocation Efficiency” of the National Social Science Foundation of China (Approval Code: 17AJL007). We also would like to thank Jason Shogren, Thorsten Janus, Sasha Skiba, Benjamin Rashford, and participants of the 2016 American Agricultural Economics Association Annual Meetings for their helpful comments and suggestions.

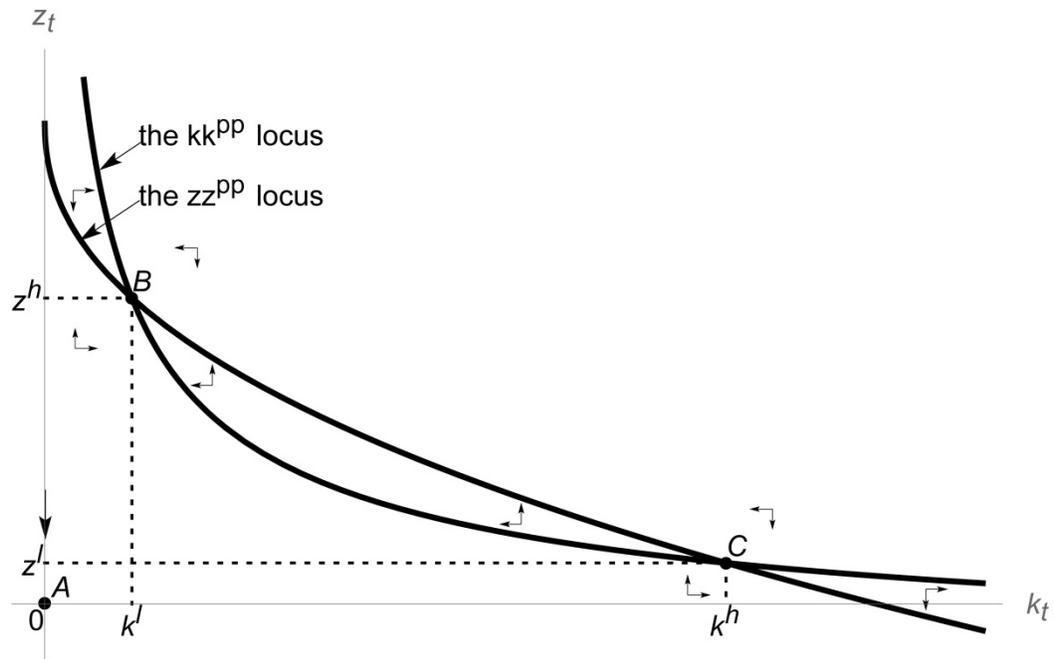


Figure 1: Phase Diagram and Multiple Equilibria under Pollution Permits

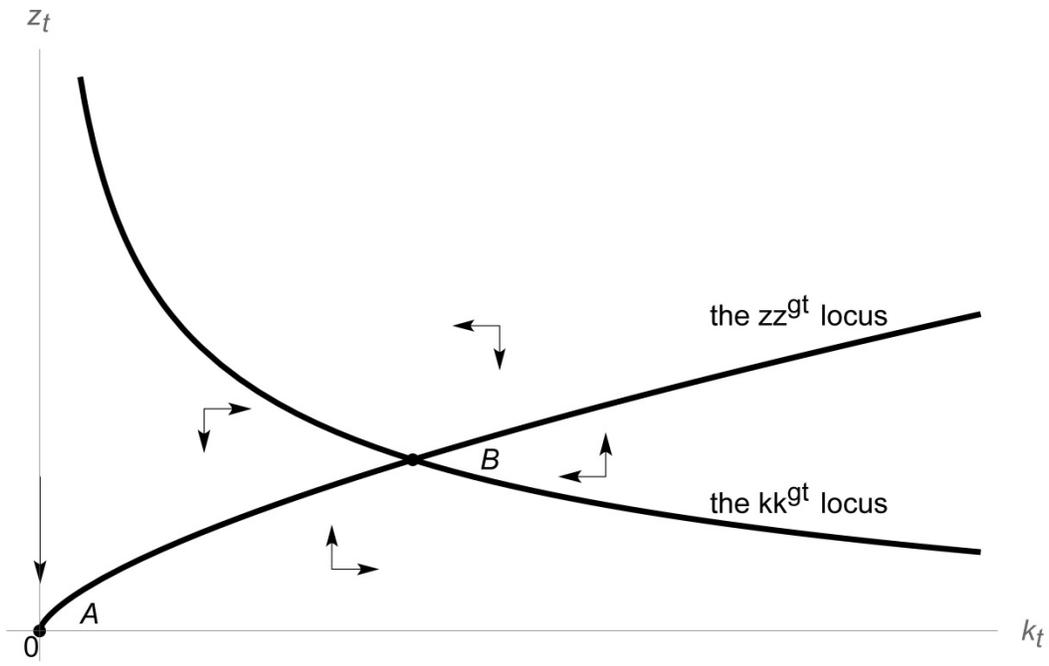


Figure 2: Phase Diagram under Green Taxes

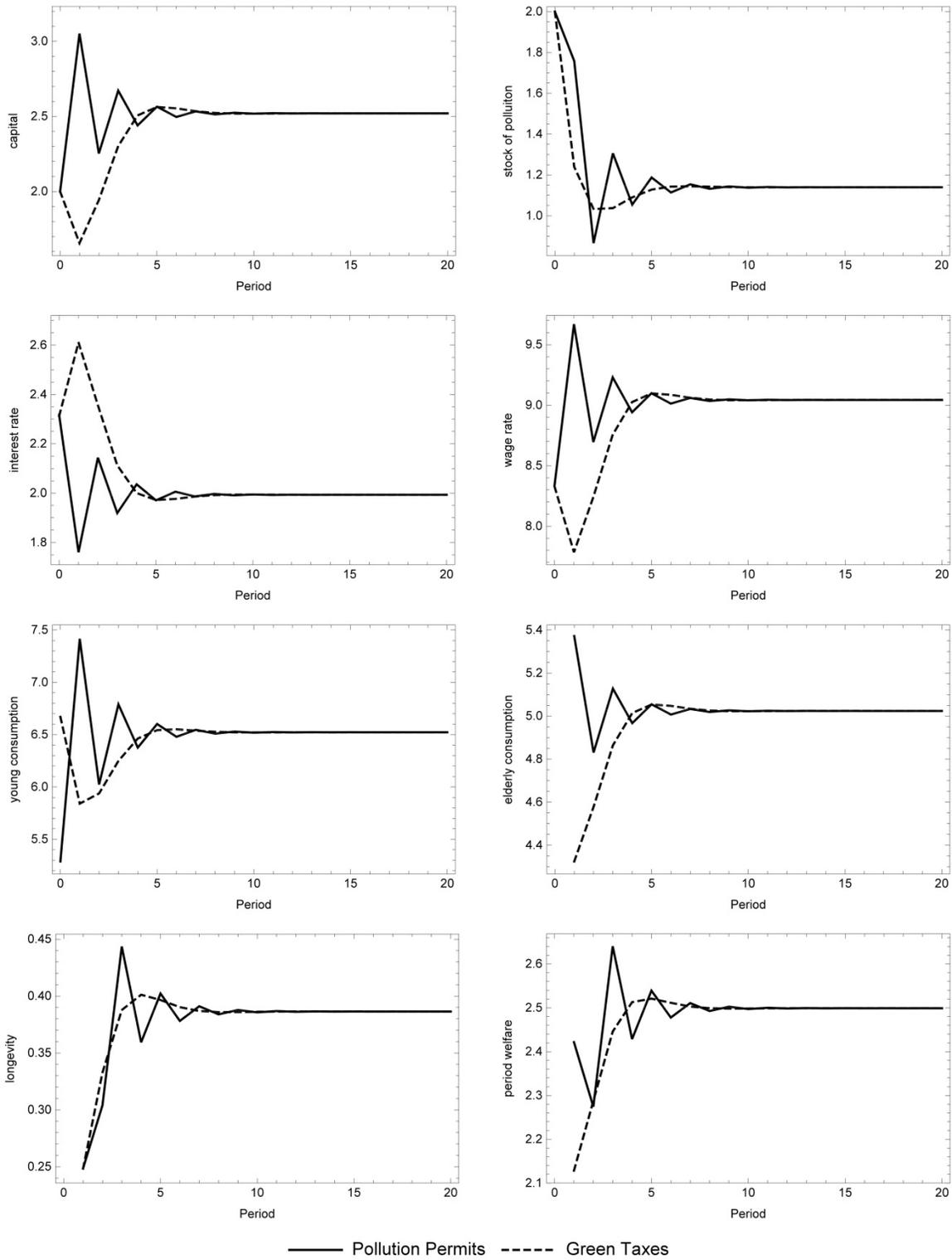


Figure 3: Comparison of Transition Paths with Government Intervention under Pollution Permits

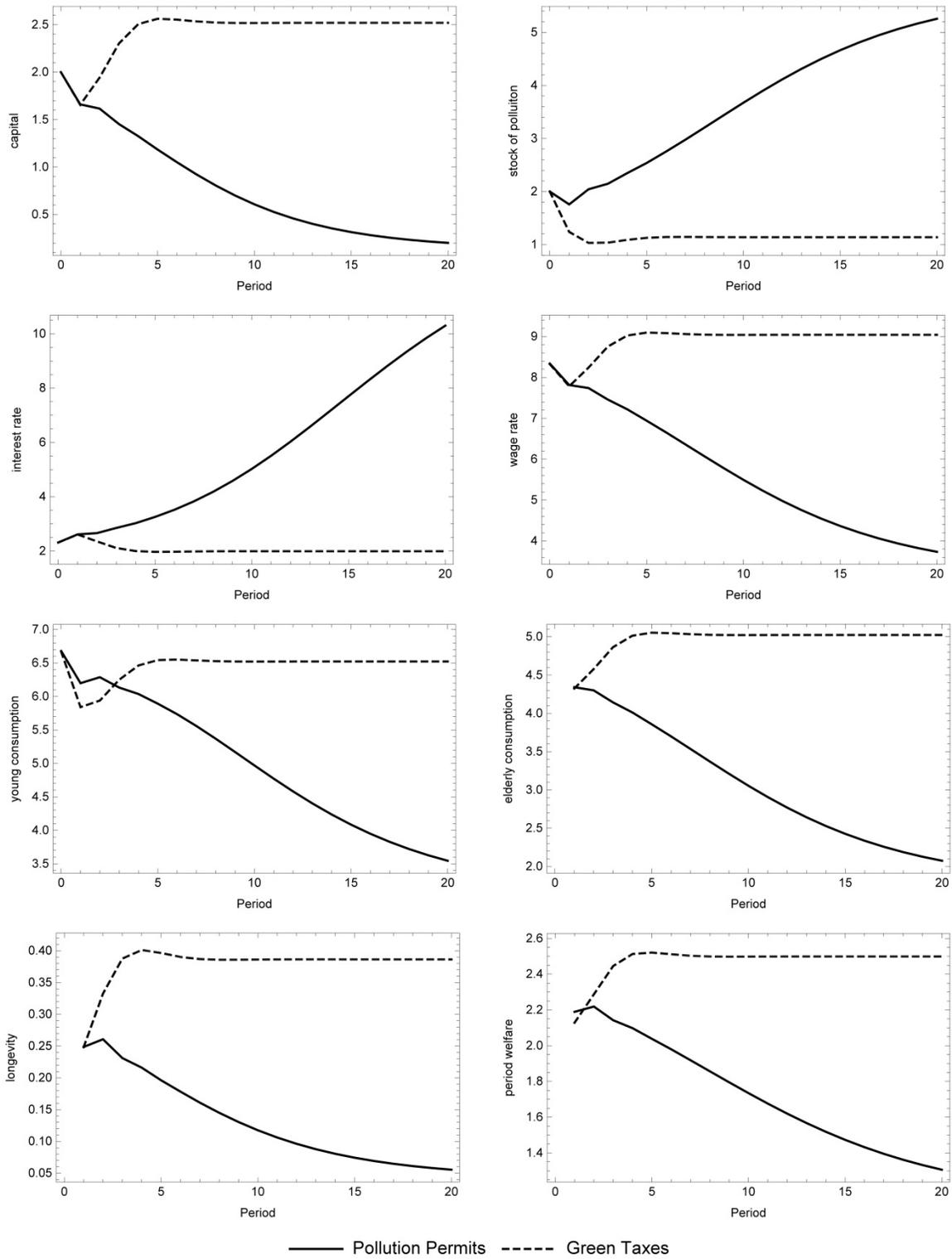


Figure 4: Comparison of Transition Paths without Government Intervention

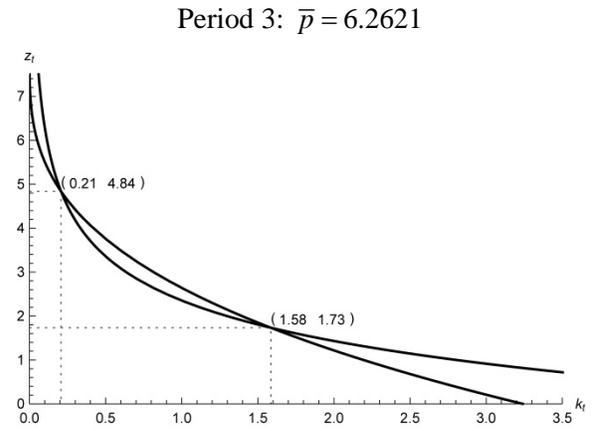
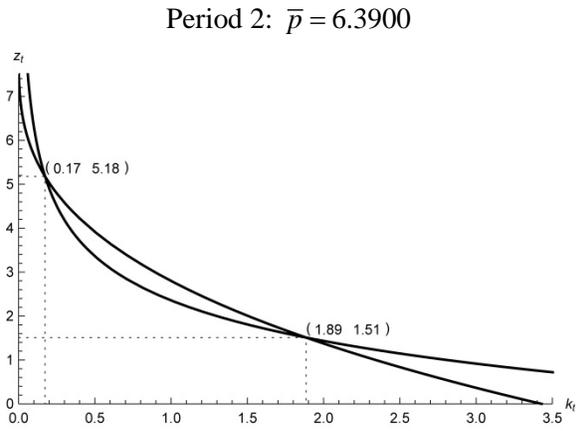
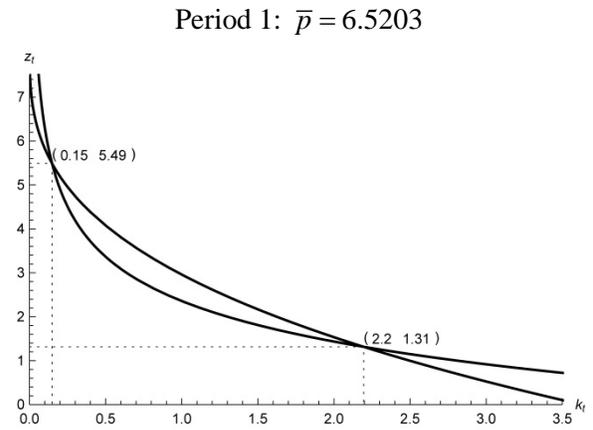
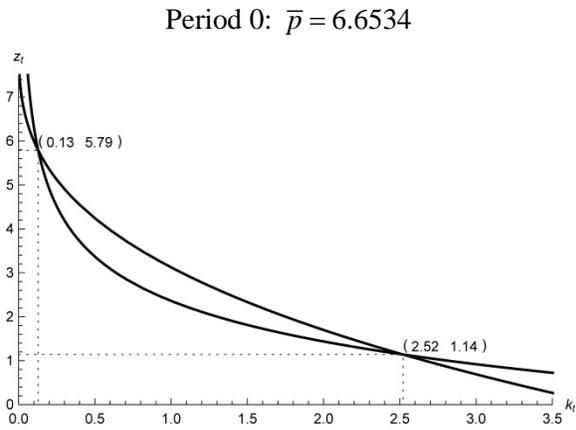


Figure 5: The Effects of Decreases in Pollution Permits (Decreases by 2% Each Period)

Table 1: Descriptions and Values for Benchmark Parameters

Description	Parameter	Value
Capital's share in production	$\alpha$	0.35*
Labor's share in production	$\beta$	0.63*
Production function scalar	$B$	10
Speed of pollution dissipation	$\theta$	0.8
Environmental maintenance efficiency	$\gamma$	20**
Lower bound of longevity	$\underline{\lambda}$	0
Upper bound of longevity	$\bar{\lambda}$	1***
Detrimental effect of pollution on health	$\eta$	20
Number of pollution permits	$\bar{p}$	6.6534****
Green-tax rate	$q$	0.0431*****
Objective discount factor	$\delta$	0.6

**Notes.** \* Brock and Taylor (2005) argue that pollution's share in the production function should be between 0.01 and 0.02. I follow the illustrative example in Brock and Taylor (2005), and set capital's share  $\alpha = 0.35$  and pollution's share  $1 - \alpha - \beta = 0.02$ , implying labor's share  $\beta = 0.63$ .

\*\* The environmental maintenance efficiency also serves as a conversion parameter that transforms the resources of environmental maintenance in the unit of final output into pollution reduction in the unit of pollution.

\*\*\* The life expectancy at birth in Japan is 83.3 years in 2013, which is the longest in the world (World Bank Group, 2015b). If each period is 50 years,  $\bar{\lambda} = 1$  implies that each generation's potential lifetime longevity is 100 years.

\*\*\*\* The value for  $\bar{p}$  is solved as the steady-state solution to the social planner's problem. The digits after the decimal point are rounded. See Appendix A for the social planner's problem.

\*\*\*\*\* The value for green-tax rate corresponds to  $\bar{p}$ . The green-tax rate is set such that the unique steady state under green taxes overlaps the desirable steady state under pollution permits. The digits after the decimal point are rounded. Also note that the parameter restriction  $1 - \gamma q > 0$  is satisfied such that a nontrivial steady state exists under green taxes.

Table 2: Steady-State Effects of Parameter Increases under Pollution Permits

Parameter	Range <sup>#</sup>	Undesirable Steady State				Desirable Steady State			
		$k_{ss}^{pp}$	$z_{ss}^{pp}$	$\phi_{ss}^{pp}$	Welfare	$k_{ss}^{pp}$	$z_{ss}^{pp}$	$\phi_{ss}^{pp}$	Welfare
$\bar{p}$	(-10%, 18%)	-*	+	-*	-*	+	-	+	+
$\gamma$	(-15%, 21%)	+	-	+	+	-	+	-	-
$\eta$	(-24%, 88%)	-	+	-	-	+	-	+	+
$\theta$	(-100%, 32%)	+	-	+	+	-	+	-	-

**Notes.** <sup>#</sup> The ranges of each parameter allow for multiple equilibria while holding other parameters constant. The ranges are expressed as percentage deviations from the benchmark values listed in Table 1.

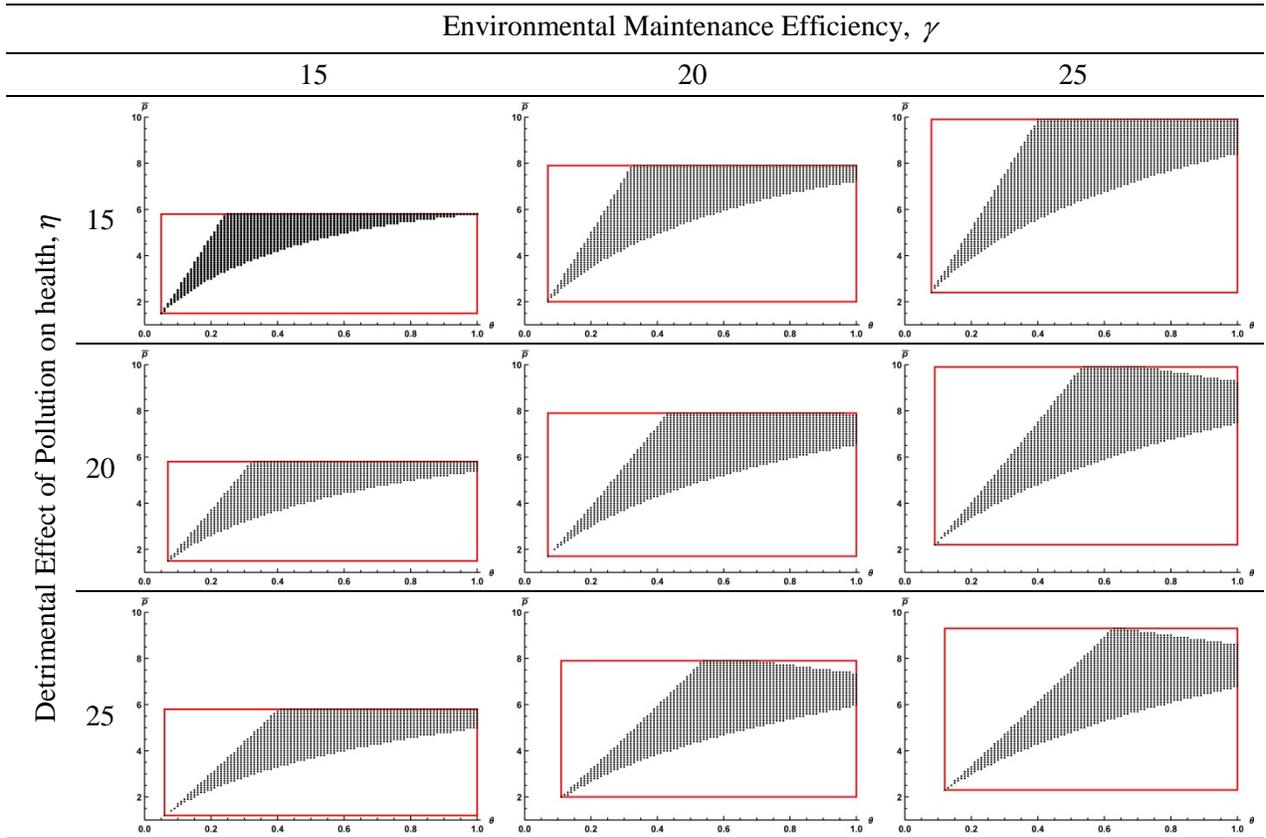
\* The signs are ambiguous in comparative statics performed in Appendix E, and are obtained using numerical simulations based on benchmark values in Table 1. The unambiguous signs derived in Appendix E can also be verified by the numerical results.

Table 3: Steady-State Effects of Parameter Increases under Green Taxes

Spirally Stable Steady State				
Parameter	$k_{ss}^{gt}$	$z_{ss}^{gt}$	$\phi_{ss}^{gt}$	Welfare
$q$	+ <sup>*</sup>	-	+ <sup>*</sup>	+ <sup>*</sup>
$\gamma$	+	-	+	+
$\eta$	-	-	- <sup>*</sup>	- <sup>*</sup>
$\theta$	+	-	+	+

**Notes.** <sup>\*</sup> The signs are ambiguous in comparative statics performed in Appendix E, and are obtained using numerical simulations based on benchmark values in Table 1. The unambiguous signs derived in Appendix E can also be verified by the numerical results.

Table 4: Sensitivity Analysis of Multiple Equilibria under Pollution Permits



**Notes.** In this  $3 \times 3$  table, each column represents a value for the environmental maintenance efficiency  $\gamma$  and each row represents a value for the detrimental effect of pollution on health  $\eta$ . In each cell of the table, the graph shows all of the combinations of the dissipation rate of pollution  $\theta$  and the number of pollution permits  $\bar{p}$  that give rise to multiple equilibria under pollution permits, with  $\theta$  on the horizontal axis and  $\bar{p}$  on the vertical axis. For example, when  $\gamma = 15$  and  $\eta = 15$  (the cell in the top-left corner of the table), the combination of  $\theta = 0.3$  and  $\bar{p} = 5$  leads to the emergence of multiple equilibria.

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# Pollution Permits, Green Taxes, and the Environmental Poverty Trap

(Supplementary Document)

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July 9, 2018

## Appendix A. The Social Planner's Problem

The production function in per-capita terms is

$$y_t = Bk_t^\alpha p_t^{1-\alpha-\beta}, \quad (\text{A.1})$$

and the lifetime utility of the agent born at the beginning of period  $t$  is

$$U_t = U(c_t, d_{t+1}, p_t, k_t, z_t) = \ln c_t + \frac{y(k_t, p_t)}{\eta z_t + y(k_t, p_t)} \ln d_{t+1}. \quad (\text{A.2})$$

In period  $t$  in per-capita terms, total output  $y_t$  becomes young consumption  $c_t$ , elderly consumption  $d_t$ , environmental maintenance  $a_t$ , and next period's physical capital  $k_{t+1}$ . The resource constraint is

$$y_t - c_t - d_t - a_t - k_{t+1} = 0. \quad (\text{A.3})$$

The dynamics for the stock of pollution is

$$z_{t+1} - (1-\theta)z_t + \gamma a_t - p_t = 0. \quad (\text{A.4})$$

With  $\delta \in (0,1)$  as the discount factor, the social planner's problem is to maximize the discounted sum of agents' utility

$$\sum_{t=0}^{+\infty} \delta^t U_t, \quad (\text{A.5})$$

subject to (A.3) and (A.4) by choosing young consumption  $c_t$ , elderly consumption  $d_t$ , environmental maintenance  $a_t$ , pollution flow  $p_t$ , as well as physical capital  $k_t$  and the stock of pollution  $z_t$ . The Lagrangian associated with the social planner's problem is

$$\ell = \sum_{t=0}^{+\infty} \delta^t \left[ U_t + \omega_{t+1} (y_t - c_t - d_t - a_t - k_{t+1}) + \nu_{t+1} (z_{t+1} - (1-\theta)z_t + \gamma a_t - p_t) \right], \quad (\text{A.6})$$

where  $\omega_{t+1}$  and  $\nu_{t+1}$  are Lagrangian multipliers of resource constraint and the stock of pollution dynamics.

Taking derivatives of  $\ell$  in (A.6) with respect to  $c_t$ ,  $d_t$ ,  $a_t$ ,  $p_t$ ,  $k_t$ , and  $z_t$ , we have

$$c_t : \frac{\partial U_t}{\partial c_t} = \omega_{t+1}, \quad (\text{A.7})$$

$$d_t : \frac{\partial U_{t+1}}{\partial d_t} = \delta \omega_{t+1}, \quad (\text{A.8})$$

$$a_t : \gamma \nu_{t+1} = \omega_{t+1}, \quad (\text{A.9})$$

$$p_t : \frac{\partial U_t}{\partial p_t} + \omega_{t+1} \frac{\partial y_t}{\partial p_t} = \nu_{t+1}, \quad (\text{A.10})$$

$$k_t : \delta \left[ \frac{\partial U_t}{\partial k_t} + \omega_{t+1} \frac{\partial y_t}{\partial k_t} \right] = \omega_t, \quad (\text{A.11})$$

$$z_t : \delta \left( (1-\theta) \nu_{t+1} - \frac{\partial U_t}{\partial z_t} \right) = \nu_t. \quad (\text{A.12})$$

Substituting (A.7) into (A.8) to eliminate  $\omega_{t+1}$  and writing at period  $t+1$  gives the tradeoff between generations:

$$\begin{aligned}\frac{\partial U_t}{\partial d_{t+1}} &= \delta \frac{\partial U_{t+1}}{\partial c_{t+1}}, \\ \frac{y_t}{\eta z_t + y_t} \frac{1}{d_{t+1}} &= \delta \frac{1}{c_{t+1}}.\end{aligned}\tag{A.13}$$

From (A.7), (A.9), and (A.10), we have the tradeoff between the flow of pollution and young consumption:

$$\begin{aligned}\frac{\partial U_t}{\partial p_t} &= \left( \frac{1}{\gamma} - \frac{\partial y_t}{\partial p_t} \right) \frac{\partial U_t}{\partial c_t}, \\ \frac{\partial y_t}{\partial p_t} \frac{\eta z_t}{(\eta z_t + y_t)^2} \ln d_{t+1} &= \left( \frac{1}{\gamma} - \frac{\partial y_t}{\partial p_t} \right) \frac{1}{c_t}.\end{aligned}\tag{A.14}$$

Rewriting (A.11) at period  $t+1$ , substituting (A.7) in to eliminate  $\omega_{t+1}$ , and substituting (A.8) in to eliminate  $\omega_{t+2}$  gives the tradeoff between consumptions over an agent's life cycle:

$$\begin{aligned}\delta \frac{\partial U_{t+1}}{\partial k_{t+1}} + \frac{\partial y_{t+1}}{\partial k_{t+1}} \frac{\partial U_t}{\partial d_{t+1}} &= \frac{\partial U_t}{\partial c_t}, \\ \frac{\partial y_{t+1}}{\partial k_{t+1}} \left[ \delta \frac{\eta z_{t+1}}{(\eta z_{t+1} + y_{t+1})^2} \ln d_{t+2} + \frac{y_t}{\eta z_t + y_t} \frac{1}{d_{t+1}} \right] &= \frac{1}{c_t}.\end{aligned}\tag{A.15}$$

Rewriting (A.12) at period  $t+1$  and substituting (A.7) and (A.9) in gives the tradeoff between consumption and the stock of pollution:

$$\begin{aligned}\delta \left[ (1-\theta) \frac{\partial U_{t+1}}{\partial c_{t+1}} - \gamma \frac{\partial U_{t+1}}{\partial z_{t+1}} \right] &= \frac{\partial U_t}{\partial c_t}, \\ \delta \left[ (1-\theta) \frac{1}{c_{t+1}} + \frac{\gamma \eta y_{t+1}}{(\eta z_{t+1} + y_{t+1})^2} \ln d_{t+2} \right] &= \frac{1}{c_t}.\end{aligned}\tag{A.16}$$

Using (A.3) and (A.4) to eliminate  $a_t$  gives

$$z_{t+1} - (1-\theta)z_t + \gamma(y_t - c_t - d_t - k_{t+1}) - p_t = 0. \quad (\text{A.17})$$

Next, we find the steady-state solution to the social planner's problem with 5 equations in steady state, (A.13)-(A.17), and 5 unknown steady-state values,  $c, d, p, k$ , and  $z$ . Substituting the entire expressions of (A.13) and (A.14) into (A.15) gives  $k$  as a function of  $p$ :

$$k = \frac{\delta}{\gamma} \frac{\alpha}{1-\alpha-\beta} p.$$

Substituting the expression of (A.14) into (A.16) gives  $z$  as a function of  $p$ :

$$z = \frac{\frac{\delta}{1-\alpha-\beta} p - \gamma \delta B \left( \frac{\delta}{\gamma} \frac{\alpha}{1-\alpha-\beta} \right)^\alpha p^{1-\beta}}{1-\delta(1-\theta)}.$$

Substituting  $k$  and  $z$  as functions of  $p$  into (A.13) and (A.17) gives two equations with  $p, c$ , and  $d$ . So we can express  $c$  and  $d$  as functions of  $p$ . Substituting  $c, d, k$ , and  $z$  as functions of  $p$  into (A.14) gives an equation with only  $p$ , which leads to the steady-state value for  $p$  and further the steady-state values for  $c, d, a, k$ , and  $z$ .

## **Appendix B. Proofs under Environmental Regulation with Pollution Permits**

In Appendix B, we prove Proposition 1 that summarizes the conditions for the emergence of multiple equilibria under pollution permits. Then we focus on the case where multiple equilibria emerge and the conditions under which multiple (dual) equilibria are satisfied. To show the local dynamic properties surrounding the two nontrivial steady states, we divide the subsequent

analysis into four parts. First, we analytically prove the slopes of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus. Second, we figure out the relative positions of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus. Third, we prove Proposition 2 based on the slopes of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus. Fourth, we check the global curvature of the  $kk^{pp}$  locus to eliminate the third possible steady state and corroborate Proposition 1.

Proof #1. Proposition 1.

Under the Assumptions (i)  $0 < \alpha < \beta < 1$  and (ii)  $\underline{\lambda} = 0$  and  $\bar{\lambda} = 1$ , substituting (23) into (20) to eliminate  $z_t$  and rearranging gives

$$\frac{1}{\eta} \beta B^2 \bar{p}^{2(1-\alpha-\beta)} = \frac{\bar{p}}{\theta} k_t^{1-2\alpha} + \left[ \frac{2}{\eta} - \frac{\gamma}{\theta} (1-\alpha-\beta) \right] B \bar{p}^{1-\alpha-\beta} k_t^{1-\alpha} \equiv T(k_t).$$

For  $T(k_t)$ ,  $T(0) = 0$  and  $T'(k_t) = (1-2\alpha) \frac{\bar{p}}{\theta} k_t^{-2\alpha} + (1-\alpha) \left[ \frac{2}{\eta} - \frac{\gamma}{\theta} (1-\alpha-\beta) \right] B \bar{p}^{1-\alpha-\beta} k_t^{-\alpha}$ . First,

when  $\frac{2}{\eta} - \frac{\gamma}{\theta} (1-\alpha-\beta) \geq 0 \Leftrightarrow \gamma \leq \frac{2}{1-\alpha-\beta} \frac{\theta}{\eta}$ ,  $T'(k_t) > 0$  for  $k_t > 0$ , and only one solution for  $k_t$

exists. In the second case,  $\frac{2}{\eta} - \frac{\gamma}{\theta} (1-\alpha-\beta) < 0 \Leftrightarrow \gamma > \frac{2}{1-\alpha-\beta} \frac{\theta}{\eta}$ . If  $0 < k_t < \left[ \frac{1-2\alpha}{1-\alpha} \frac{\bar{p}^{\alpha+\beta}}{[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}]B} \right]^{\frac{1}{\alpha}}$ ,

$T'(k_t) > 0$ ; if  $k_t = \left[ \frac{1-2\alpha}{1-\alpha} \frac{\bar{p}^{\alpha+\beta}}{[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}]B} \right]^{\frac{1}{\alpha}}$ ,  $T'(k_t) = 0$ ; if  $k_t > \left[ \frac{1-2\alpha}{1-\alpha} \frac{\bar{p}^{\alpha+\beta}}{[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}]B} \right]^{\frac{1}{\alpha}}$ ,  $T'(k_t) < 0$ . So the

function  $T(k_t)$  first rises, reaches its peak at  $k_t = \left[ \frac{1-2\alpha}{1-\alpha} \frac{\bar{p}^{\alpha+\beta}}{[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}]B} \right]^{\frac{1}{\alpha}} \equiv k^*$ , and then falls. So the

emergence of multiple equilibria also requires that  $\frac{1}{\eta} \beta B^2 \bar{p}^{2(1-\alpha-\beta)}$  must be lower than the peak of

$T(k_t)$ , i.e.,  $T(k^*)$ .

Next, we focus on the case where multiple equilibria emerge. We will show that without introducing additional assumptions other than Assumptions (i) and (ii), the following facts can be established.

Proof #2. The  $kk^{pp}$  locus and the  $zz^{pp}$  locus slope down in  $(k_t, z_t)$  space.

Start from equations (20) and (23):

$$\Phi(y^{pp}(k_t, \bar{p}), z_t)w^{pp}(k_t, \bar{p}) - k_t = 0, \quad (\text{B.1})$$

$$-\theta z_t - \gamma a^{pp}(k_t, \bar{p}) + \bar{p} = 0. \quad (\text{B.2})$$

Now we check the slope of the  $kk^{pp}$  locus. Totally differentiating (B.1) and rearranging gives

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } kk^{pp} \text{ locus}} = - \frac{\frac{\partial \Phi_{t+1}}{\partial y_t^{pp}} \frac{\partial y_t^{pp}}{\partial k_t} w_t^{pp} + \Phi_{t+1} \frac{\partial w_t^{pp}}{\partial k_t} - 1}{\frac{\partial \Phi_{t+1}}{\partial z_t} w_t^{pp}}. \quad (\text{B.3})$$

Substituting (B.1) into (B.3) and rewriting gives

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } kk^{pp} \text{ locus}} = - \frac{\left( \frac{\partial \Phi_{t+1}}{\partial y_t^{pp}} \frac{y_t^{pp}}{\Phi_{t+1}} \right) \left( \frac{\partial y_t^{pp}}{\partial k_t} \frac{k_t}{y_t^{pp}} \right) + \left( \frac{\partial w_t^{pp}}{\partial k_t} \frac{k_t}{w_t^{pp}} \right) - 1}{\left( \frac{\partial \Phi_{t+1}}{\partial z_t} \frac{z_t}{\Phi_{t+1}} \right) \frac{k_t}{z_t}}. \quad (\text{B.4})$$

Define  $E_{\Phi_{t+1}, y_t^{pp}} = \frac{\partial \Phi_{t+1}}{\partial y_t^{pp}} \frac{y_t^{pp}}{\Phi_{t+1}}$  as the elasticity of the propensity to save  $\Phi_{t+1}$  with respect to income

per capita  $y_t^{pp}$ , and  $E_{\Phi_{t+1}, z_t} = \frac{\partial \Phi_{t+1}}{\partial z_t} \frac{z_t}{\Phi_{t+1}}$  as the elasticity of the propensity to save  $\Phi_{t+1}$  with respect

to the stock of pollution  $z_t$ . The elasticities are contained in the ranges  $E_{\Phi_{t+1}, y_t^{pp}} \in [0, 1]$  and

$E_{\Phi_{t+1}, z_t} \in [-1, 0]$ . The ranges for both of these elasticities can be verified by substituting (10) into

$\Phi_{t+1}$ . It also can be verified that  $E_t^{pp} = E_{\Phi_{t+1}, y_t^{pp}} = -E_{\Phi_{t+1}, z_t}$ , an important fact that will be used later.

Lastly,  $E_{w_t^{pp}, k_t} = \frac{\partial w_t^{pp}}{\partial k_t} \frac{k_t}{w_t^{pp}}$  is the elasticity of wage rate with respect to capital,  $E_{y_t^{pp}, k_t} = \frac{\partial y_t^{pp}}{\partial k_t} \frac{k_t}{y_t^{pp}}$  is the elasticity of income per capita with respect to capital, and both are equal to capital's share in production  $\alpha$ . Thus, (B.4) can be rewritten as

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } kk^{pp} \text{ locus}} = \left( \alpha - \frac{1-\alpha}{E_t^{pp}} \right) \frac{z_t}{k_t} \quad (\text{B.5})$$

The sufficient condition for  $dz_t/dk_t|_{\text{the } kk^{pp} \text{ locus}} < 0$  is Assumption (i)  $0 < \alpha < \beta < 1$ , which combined with  $1 - \alpha - \beta > 0$  implies that  $0 < \alpha < 1/2$ . So as long as capital's share in production is less than labor's share, the  $kk^{pp}$  locus always slopes down in  $(k_t, z_t)$  space.

Now we check the slope of the  $zz^{pp}$  locus. Totally differentiating (B.2) and using (5), we get

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } zz^{pp} \text{ locus}} = -\frac{\alpha\gamma}{\theta} \frac{a_t^{pp}}{k_t}. \quad (\text{B.6})$$

It is straightforward that  $dz_t/dk_t|_{\text{the } zz^{pp} \text{ locus}} < 0$  from (B.6), suggesting that the  $zz^{pp}$  locus slopes down in  $(k_t, z_t)$  space.

**Proof #3.** The relative positions of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus in  $(k_t, z_t)$  space.

On the  $kk^{pp}$  locus,  $k_t$  asymptotically converges to  $\left(\frac{\lambda}{\lambda+1} \beta B \bar{p}^{1-\alpha-\beta}\right)^{\frac{1}{1-\alpha}}$  when  $z_t \rightarrow +\infty$ . This fact can be established by using (3), (18), and (20). As  $z_t \rightarrow +\infty$ , the health status  $x_t \rightarrow 0$ . So the propensity to save converges to its lower bound  $\frac{\lambda}{\lambda+1}$ , and physical capital converges to  $\left(\frac{\lambda}{\lambda+1} \beta B \bar{p}^{1-\alpha-\beta}\right)^{\frac{1}{1-\alpha}}$ . On the  $zz^{pp}$  locus, in contrast, when  $k_t = 0$ ,  $z_t = \bar{p}/\theta$  by (23). So when  $k_t$  is

relatively small, the  $kk^{pp}$  locus lies above the  $zz^{pp}$  locus in  $(k_t, z_t)$  space. Combining the fact that both loci monotonically decrease in  $k_t$ , we conclude that as  $k_t$  increase, the  $kk^{pp}$  locus must first intersect with the  $zz^{pp}$  locus from above, indicating that at the steady state  $(k^l, z^h)$ , the slope of the  $kk^{pp}$  locus must be steeper than that of the  $zz^{pp}$  locus. It follows that as  $k_t$  increases until the two loci intersect again at  $(k^h, z^l)$ , the slope of the  $kk^{pp}$  locus must be flatter than that of the  $zz^{pp}$  locus.

Next, we analytically prove Proposition 2 by showing that the type of (local) transition dynamics is stable around a steady state where the slope of the  $kk^{pp}$  locus is steeper than that of the  $zz^{pp}$  locus, while a steady state where the slope of the  $kk^{pp}$  locus is flatter than that of the  $zz^{pp}$  locus can be unstable or exhibits saddle-path stability.<sup>1</sup>

#### Proof #4. Proposition 2

Rewrite equations (18) and (21) as

$$k_{t+1} = \Phi(y^{pp}(k_t), z_t) w^{pp}(k_t), \quad (\text{B.7})$$

$$z_{t+1} = (1 - \theta)z_t - \gamma a^{pp}(k_t) + \bar{p}. \quad (\text{B.8})$$

Then, totally differentiate (B.7) and (B.8) around the steady states to produce

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<sup>1</sup> Steady state ‘C’ in Figure 1 from the main paper exhibits saddle-path stability. Any initial levels of capital to the left of the saddle path will lead to a path transitioning into the undesirable steady state ‘B’. Initial levels of capital to the right of the saddle path will lead to a path that eventually violates the condition  $z \geq 0$ . We have ruled out such paths. However, we acknowledge that it may be possible to modify the model such that corner solutions are allowed where  $z = 0$ , and any excess government revenues are either returned to individuals or spent on other government programs. Despite this possible modification, the main results in this paper, i.e., the emergence of an EPT under pollution permits and the policies required to avoid it, do not change. We leave such a possibility for future research.

$$\begin{aligned}
dk_{t+1} &= \left[ \frac{\partial \Phi}{\partial y^{pp}} \frac{\partial y^{pp}}{\partial k} w^{pp} + \Phi \frac{\partial w^{pp}}{\partial k} \right] dk_t + \frac{\partial \Phi}{\partial z} w^{pp} dz_t, \\
dz_{t+1} &= -\gamma \frac{\partial a^{pp}}{\partial k} dk_t + (1-\theta) dz_t,
\end{aligned}$$

where the subscripts  $t$  of functions and variables are all left out to indicate that the partial derivatives are evaluated at the steady state, either at  $(k^l, z^h)$  or at  $(k^h, z^l)$ . The associated Jacobian matrix is

$$J^{pp} = \begin{pmatrix} \frac{\partial \Phi}{\partial y^{pp}} \frac{\partial y^{pp}}{\partial k} w^{pp} + \Phi \frac{\partial w^{pp}}{\partial k} & \frac{\partial \Phi}{\partial z} w^{pp} \\ -\gamma \frac{\partial a^{pp}}{\partial k} & 1-\theta \end{pmatrix}. \quad (\text{B.9})$$

Next, substituting (B.1) into (B.9) and rearranging gives

$$J^{pp} = \begin{pmatrix} \alpha(E^{pp} + 1) & -E^{pp} \frac{k}{z} \\ -\gamma \alpha \frac{a^{pp}}{k} & 1-\theta \end{pmatrix}, \quad (\text{B.10})$$

where  $E_{w^{pp},k} = \frac{\partial w^{pp}}{\partial k} \frac{k}{w^{pp}} = \alpha$  is the elasticity of the wage rate with respect to capital,

$E_{y^{pp},k} = \frac{\partial y^{pp}}{\partial k} \frac{k}{y^{pp}} = \alpha$  is the elasticity of the income per capita with respect to capital,

$E_{\Phi,y^{pp}} = \frac{\partial \Phi}{\partial y^{pp}} \frac{y^{pp}}{\Phi} = E^{pp}$  is the elasticity of the propensity to save with respect to income per capita,

and  $E_{\Phi,z} = \frac{\partial \Phi}{\partial z} \frac{z}{\Phi} = -E^{pp}$  is the elasticity of the propensity to save with respect to the stock of pollution. All the elasticities are evaluated at the steady state.

The trace and determinant of the Jacobian matrix are

$$\begin{aligned} TrJ^{pp} &= \alpha(E^{pp} + 1) + (1 - \theta) < 2, \\ DeJ^{pp} &= (1 - \theta)\alpha(E^{pp} + 1) - \alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp} < 1. \end{aligned}$$

Because  $E^{pp} \in [0, 1]$  and  $\theta \in (0, 1]$ , both inequalities must hold when  $\alpha \in (0, 1/2)$ . Further, it is straightforward to verify that the eigenvalues are real and distinct because

$$\begin{aligned} (TrJ^{pp})^2 - 4DeJ^{pp} &= \left[\alpha(E^{pp} + 1) + (1 - \theta)\right]^2 - 4(1 - \theta)\alpha(E^{pp} + 1) + 4\alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp} \\ &= \left[\alpha(E^{pp} + 1) - (1 - \theta)\right]^2 + 4\alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp} > 0. \end{aligned}$$

The characteristic polynomial is  $p(v) = v^2 - (TrJ^{pp})v + DeJ^{pp}$ , and

$$\begin{aligned} p(1) &= 1 - TrJ^{pp} + DeJ^{pp} = \alpha\theta\left(\frac{1 - \alpha}{\alpha} - \frac{\bar{p}}{\theta z}E^{pp}\right), \\ p(-1) &= 1 + TrJ^{pp} + DeJ^{pp} = \alpha\theta\left[\left(\frac{2}{\theta} - 1\right)\frac{1 + \alpha}{\alpha} + \left(\frac{2}{\theta} - \frac{\bar{p}}{\theta z}\right)E^{pp}\right], \end{aligned}$$

where both expressions make use of (B.2).

For the steady state  $(k^l, z^h)$  where the slope of the  $kk^{pp}$  locus is steeper than that of the  $zz^{pp}$  locus, by (B.5) and (B.6), the condition is

$$\left(\alpha - \frac{1 - \alpha}{E^{pp}}\right)\frac{z}{k} < -\frac{\alpha\gamma}{\theta}\frac{a^{pp}}{k}. \quad (\text{B.11})$$

Rearranging (B.11) and using (B.2) gives

$$\frac{1 - \alpha}{\alpha} - \frac{\bar{p}}{\theta z}E^{pp} > 0. \quad (\text{B.12})$$

From (B.12), it is straightforward that  $p(1) > 0$ . Because

$$p(-1) = \alpha\theta \left[ \left( \frac{2}{\theta} - 1 \right) \frac{1+\alpha}{\alpha} + \left( \frac{2}{\theta} - \frac{\bar{p}}{\theta z} \right) E^{pp} \right] > \alpha\theta \left( \frac{1-\alpha}{\alpha} - \frac{\bar{p}}{\theta z} E^{pp} \right) = p(1) > 0,$$

so  $p(-1) > 0$ . Further, it is already shown that  $DeJ^{pp} < 1$ . We thus conclude that the two eigenvalues associated with the steady state  $(k^l, z^h)$  lie in the interval  $(-1, 1)$ , and the steady state  $(k^l, z^h)$  is a stable equilibrium.

In contrast, for the steady state  $(k^h, z^l)$  where the slope of the  $kk^{pp}$  locus is flatter than that of the  $zz^{pp}$  locus, by (B.5) and (B.6), the condition is

$$\left( \alpha - \frac{1-\alpha}{E^{pp}} \right) \frac{z}{k} > -\frac{\alpha\gamma}{\theta} \frac{a^{pp}}{k}. \quad (\text{B.13})$$

Rearranging gives

$$\frac{1-\alpha}{\alpha} - \frac{\bar{p}}{\theta z} E^{pp} < 0, \quad (\text{B.14})$$

so  $p(1) < 0$ . For  $p(-1)$ , because  $p(-1) > p(1)$ , there are two possible cases. If the steady-state stock of pollution is sufficiently large, i.e.  $z^l > \bar{p} / \left( \frac{1+\alpha}{\alpha} \frac{2-\theta}{E^{pp}} + 2 \right)$ , where  $E^{pp}$  is the elasticity evaluated at the steady state  $(k^h, z^l)$ , so  $p(-1) > 0$ . One eigenvalue lies between  $(-1, 1)$ , while the other is greater than 1, indicating that the steady state  $(k^h, z^l)$  is a saddle equilibrium. However, if the steady-state stock of pollution is sufficiently small, i.e.,  $z^l < \bar{p} / \left( \frac{1+\alpha}{\alpha} \frac{2-\theta}{E^{pp}} + 2 \right)$ , where  $E^{pp}$  is the elasticity evaluated at the steady state  $(k^h, z^l)$ , so  $p(-1) < 0$ . Since one eigenvalue is greater than 1 and the other is smaller than -1, the steady state  $(k^h, z^l)$  is not stable.

Proof #5. The global curvature of the  $kk^{pp}$  locus in  $(k_t, z_t)$  space.

We now check the curvature of the  $kk^{pp}$  locus. The reason for doing this is that if the  $kk^{pp}$  locus is concave for some range of  $k_t$  while convex for another range, more than two non-trivial steady states may emerge and the desirable steady state may be stable rather than saddle. Note that  $E_t^{pp} = E(y^{pp}(k_t, \bar{p}), z_t)$  and  $z_t = z(k_t)$ . Differentiating (B.5) with respect to  $k_t$  gives

$$\left. \frac{d^2 z_t}{dk_t^2} \right|_{\text{the } kk^{pp} \text{ locus}} = \frac{1-\alpha}{(E_t^{pp})^2} \left( \frac{\partial E_t^{pp}}{\partial y_t^{pp}} \frac{\partial y_t^{pp}}{\partial k_t} + \frac{\partial E_t^{pp}}{\partial z_t} \frac{\partial z_t}{\partial k_t} \right) \frac{z_t}{k_t} + \left( \alpha - \frac{1-\alpha}{E_t^{pp}} \right) \left( \frac{\partial z_t}{\partial k_t} \frac{1}{k_t} - \frac{z_t}{k_t^2} \right). \quad (\text{B.15})$$

Substituting (B.5) into (B.15) to eliminate  $\partial z_t / \partial k_t$  and rewriting gives

$$\begin{aligned} \left. \frac{d^2 z_t}{dk_t^2} \right|_{\text{the } kk^{pp} \text{ locus}} &= \frac{1-\alpha}{(E_t^{pp})^2} \left[ \left( \frac{\partial E_t^{pp}}{\partial y_t^{pp}} \frac{y_t^{pp}}{E_t^{pp}} \right) \left( \frac{\partial y_t^{pp}}{\partial k_t} \frac{k_t}{y_t^{pp}} \right) \frac{E_t^{pp}}{k_t} + \left( \frac{\partial E_t^{pp}}{\partial z_t} \frac{z_t}{E_t^{pp}} \right) \left( \alpha - \frac{1-\alpha}{E_t^{pp}} \right) \frac{E_t^{pp}}{k_t} \right] \frac{z_t}{k_t} \\ &+ \left( \alpha - \frac{1-\alpha}{E_t^{pp}} \right) \left( \alpha - \frac{1-\alpha}{E_t^{pp}} - 1 \right) \frac{z_t}{k_t^2}. \end{aligned} \quad (\text{B.16})$$

Intensive math shows that  $\tilde{E}_t^{pp} = \frac{\partial E_t^{pp}}{\partial y_t^{pp}} \frac{y_t^{pp}}{E_t^{pp}} = -\frac{\partial E_t^{pp}}{\partial z_t} \frac{z_t}{E_t^{pp}}$ , where  $\tilde{E}_t^{pp}$  is the elasticity's elasticity. Also note that  $\frac{\partial y_t^{pp}}{\partial k_t} \frac{k_t}{y_t^{pp}} = \alpha$ . Substituting these expressions into (B.16) and rearranging gives

$$\left. \frac{d^2 z_t}{dk_t^2} \right|_{\text{the } kk^{pp} \text{ locus}} = -\frac{(1-\alpha)z_t}{(E_t^{pp} k_t)^2} \left[ \alpha (E_t^{pp})^2 - (1-2\alpha)E_t^{pp} - (1-\alpha)(\tilde{E}_t^{pp} + 1) \right]. \quad (\text{B.17})$$

Under Assumption (ii), the lower bound of longevity is zero, i.e.,  $\underline{\lambda} = 0$ . The relationship between the elasticity  $E_t^{pp}$  and the elasticity's elasticity  $\tilde{E}_t^{pp}$  collapses to  $\tilde{E}_t^{pp} + 1 = E_t^{pp}$  and (B.17) collapses to  $\left. \frac{d^2 z_t}{dk_t^2} \right|_{\text{the } kk^{pp} \text{ locus}} = -\frac{\alpha(1-\alpha)z_t}{E_t^{pp} k_t^2} \left[ E_t^{pp} - \frac{2-3\alpha}{\alpha} \right]$ . Because  $E_t^{pp} \in [0, 1]$ , as long as capital's share

in production is less than labor's share,  $\left. \frac{d^2 z_t}{dk_t^2} \right|_{\text{the } zz^{pp} \text{ locus}} > 0$ , and the conditions for the  $kk^{pp}$  locus to

be decreasing and convex always hold. Because the  $kk^{pp}$  locus is globally convex, as  $k_t$  increases, the third intersection of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus is not possible. So the steady state  $(k^h, z^l)$ , which is determined by the second intersection of the two loci, must be the desirable equilibrium.

### Appendix C. Proofs under Environmental Regulation with Green Taxes

Proof #1. The  $kk^{st}$  locus slopes down while the  $zz^{st}$  locus slopes up in  $(k_t, z_t)$  space.

Start from equations (25) and (27):

$$\Phi(y^{st}(k_t, q), z_t)w^{st}(k_t, q) - k_t = 0, \quad (C.1)$$

$$-\theta z_t + (1 - \gamma q)p^{st}(k_t, q) = 0. \quad (C.2)$$

Now we check the slope of the  $kk^{st}$  locus. Totally differentiating (C.1) and rearranging gives

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } kk^{st} \text{ locus}} = - \frac{\frac{\partial \Phi_{t+1}}{\partial y_t^{st}} \frac{\partial y_t^{st}}{\partial k_t} w_t^{st} + \Phi_{t+1} \frac{\partial w_t^{st}}{\partial k_t} - 1}{\frac{\partial \Phi_{t+1}}{\partial z_t} w_t^{st}}. \quad (C.3)$$

Substituting (C.1) into (C.3) and rearranging gives

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } kk^{st} \text{ locus}} = \frac{\alpha E_t^{st} - \beta \frac{z_t}{k_t}}{(\alpha + \beta) E_t^{st}}, \quad (C.4)$$

where  $E_{\Phi_{t+1}, y_t^{st}} = \frac{\partial \Phi_{t+1}}{\partial y_t^{st}} \frac{y_t^{st}}{\Phi_{t+1}}$  is the elasticity of the propensity to save  $\Phi_{t+1}$  with respect to income per capita  $y_t^{st}$ , and  $E_{\Phi_{t+1}, z_t} = \frac{\partial \Phi_{t+1}}{\partial z_t} \frac{z_t}{\Phi_{t+1}}$  is the elasticity of the propensity to save  $\Phi_{t+1}$  with respect to the stock of pollution  $z_t$ . The elasticities are contained in the ranges  $E_{\Phi_{t+1}, y_t^{st}} \in [0, 1]$  and

$E_{\Phi_{t+1}, z_t} \in [-1, 0]$ . Again, it can be verified that  $E_t^{gt} = E_{\Phi_{t+1}, y_t^{gt}} = -E_{\Phi_{t+1}, z_t}$ . Lastly,  $E_{w_t^{gt}, k_t} = \frac{\partial w_t^{gt}}{\partial k_t} \frac{k_t}{w_t^{gt}}$  is the elasticity of the wage rate with respect to capital,  $E_{y_t^{gt}, k_t} = \frac{\partial y_t^{gt}}{\partial k_t} \frac{k_t}{y_t^{gt}}$  is the elasticity of income per capita with respect to capital, and both are equal to capital's share in production  $\alpha/(\alpha + \beta)$  by (1), (3), and (4). The sufficient condition for the  $kk^{gt}$  locus to slope down in  $(k_t, z_t)$  space by (C.4) is Assumption (i)  $0 < \alpha < \beta < 1$ .

Next, we check the slope of the  $zz^{gt}$  locus. Totally differentiating (C.2), using (4), and rearranging gives

$$\left. \frac{dz_t}{dk_t} \right|_{\text{the } zz^{gt} \text{ locus}} = \frac{1 - \gamma q}{\theta} \frac{\alpha}{\alpha + \beta} \frac{p_t^{gt}}{k_t} > 0. \quad (\text{C.5})$$

Thus, the  $zz^{gt}$  locus slopes up in  $(k_t, z_t)$  space.

Proof #2. Proposition 3.

Rewrite equations (24) and (26) as

$$k_{t+1} = \Phi(y^{gt}(k_t), z_t) w^{gt}(k_t), \quad (\text{C.6})$$

$$z_{t+1} = (1 - \theta)z_t + (1 - \gamma q)p^{gt}(k_t). \quad (\text{C.7})$$

Totally differentiating (C.6) and (C.7) around the steady state gives

$$dk_{t+1} = \left[ \frac{\partial \Phi}{\partial y^{gt}} \frac{\partial y^{gt}}{\partial k} w^{gt} + \Phi \frac{\partial w^{gt}}{\partial k} \right] dk_t + \frac{\partial \Phi}{\partial z} w^{gt} dz_t,$$

$$dz_{t+1} = (1 - \gamma q) \frac{\partial p^{gt}}{\partial k} dk_t + (1 - \theta) dz_t.$$

Again, the  $t$  subscripts are omitted to indicate that the derivatives are evaluated at the steady state. The associated Jacobian matrix is

$$J^{st} = \begin{pmatrix} \frac{\partial \Phi}{\partial y^{st}} \frac{\partial y^{st}}{\partial k} w^{st} + \Phi \frac{\partial w^{st}}{\partial k} & \frac{\partial \Phi}{\partial z} w^{st} \\ (1-\gamma q) \frac{\partial p^{st}}{\partial k} & 1-\theta \end{pmatrix}. \quad (\text{C.8})$$

Substituting (1), (3), (4), (C.1) and (C.2) evaluated at the steady state into (C.8) and rearranging gives

$$J^{st} = \begin{pmatrix} \frac{\alpha}{\alpha + \beta} (E^{st} + 1) & -E^{st} \frac{k}{z} \\ \theta \frac{\alpha}{\alpha + \beta} \frac{z}{k} & 1 - \theta \end{pmatrix}, \quad (\text{C.9})$$

where  $E_{w^{st},k} = \frac{\partial w^{st}}{\partial k} \frac{k}{w^{st}} = \frac{\alpha}{\alpha + \beta}$  is the elasticity of the wage rate with respect to capital,

$E_{y^{st},k} = \frac{\partial y^{st}}{\partial k} \frac{k}{y^{st}} = \frac{\alpha}{\alpha + \beta}$  is the elasticity of income per capita with respect to capital,

$E_{\Phi, y^{st}} = \frac{\partial \Phi}{\partial y^{st}} \frac{y^{st}}{\Phi} = E^{st}$  is the elasticity of the propensity to save with respect to income per capita,

and  $E_{\Phi, z} = \frac{\partial \Phi}{\partial z} \frac{z}{\Phi} = -E^{st}$  is the elasticity of the propensity to save with respect to the stock of pollution. All the elasticities are evaluated at the steady state. The trace and determinant of the Jacobian matrix are

$$TrJ^{st} = \frac{\alpha}{\alpha + \beta} (E^{st} + 1) + (1 - \theta) < 2, \quad (\text{C.10})$$

$$DeJ^{st} = \frac{\alpha}{\alpha + \beta} [(1 - \theta) + E^{st}] < 1. \quad (\text{C.11})$$

The sufficient condition for both inequalities (C.10) and (C.11) to hold is Assumption (i)

$0 < \alpha < \beta < 1$ , which implies that  $0 < \alpha/(\alpha + \beta) < 1/2$ . Under this assumption, the inequalities

(C.10) and (C.11) must hold because  $E^{gt} \in [0,1]$  and  $\theta \in (0,1]$ . Further, we need to check the sign of the term  $(TrJ^{gt})^2 - 4DeJ^{gt}$ , which establishes whether the eigenvalues have imaginary parts. From (C.10) and (C.11), we have

$$\begin{aligned} (TrJ^{gt})^2 - 4DeJ^{gt} &= \left[ \frac{\alpha}{\alpha + \beta} (E^{gt} + 1) + (1 - \theta) \right]^2 - 4 \frac{\alpha}{\alpha + \beta} [(1 - \theta) + E^{gt}] \\ &= \left[ \frac{\alpha}{\alpha + \beta} (E^{gt} + 1) - (1 - \theta) \right]^2 - 4\theta \frac{\alpha}{\alpha + \beta} E^{gt}. \end{aligned}$$

There are two possible cases. First, if  $E^{gt}$ , evaluated at the unique steady state, satisfies  $\left[ \frac{\alpha}{\alpha + \beta} (E^{gt} + 1) - (1 - \theta) \right]^2 - 4\theta \frac{\alpha}{\alpha + \beta} E^{gt} < 0$ , so  $(TrJ^{gt})^2 - 4DeJ^{gt} < 0$ , implying that the eigenvalues are complex conjugates. With  $DeJ^{gt} < 1$ , we conclude that the system under green taxes will converge to the unique steady state and the convergence is cyclical. Second, if  $E^{gt}$ , evaluated at the unique steady state, satisfies  $\left[ \frac{\alpha}{\alpha + \beta} (E^{gt} + 1) - (1 - \theta) \right]^2 - 4\theta \frac{\alpha}{\alpha + \beta} E^{gt} > 0$ , so  $(TrJ^{gt})^2 - 4DeJ^{gt} > 0$ , implying that the eigenvalues are real and distinct. The characteristic polynomial is  $p(v) = v^2 - (TrJ^{gt})v + DeJ^{gt}$ , and

$$\begin{aligned} p(1) &= 1 - TrJ^{gt} + DeJ^{gt} = \frac{\theta\beta}{\alpha + \beta} > 0, \\ p(-1) &= 1 + TrJ^{gt} + DeJ^{gt} = (2 - \theta) \frac{2\alpha + \beta}{\alpha + \beta} + \frac{2\alpha}{\alpha + \beta} E^{gt} > 0. \end{aligned}$$

With  $TrJ^{gt} < 2$  and  $DeJ^{gt} < 1$ , the system under green taxes will converge to the unique steady state and the convergence is non-cyclical.

## Appendix D. An Alternative Model with Private Healthcare

In the basic model, the representative agent treats her longevity as given. Following Bhattacharya and Qiao (2007) and Raffin and Seegmuller (2017), we consider an alternative model in which the representative agent actively engages in private healthcare efforts to improve her health status and prolong her longevity. Health status is now specified as

$$x_t = x(e_t) = \frac{e_t^\mu y_t^{1-\mu}}{\eta z_t}, \quad (\text{D.1})$$

where  $e_t$  is private healthcare expenditure,  $y_t$  is income per capita,  $z_t$  is the stock of pollution,  $\eta$  measures the detrimental effect of pollution on the health status, and  $\mu \in (0,1)$  is a positive parameter. Note that when the representative agent chooses private healthcare expenditure, she takes income per capita and the stock of pollution as given. The representative agent's longevity function is still given by equation (9).

To maintain tractability of the model, we assume the representative agent born in period  $t$  derives utility from elderly consumption  $d_{t+1}$  only. This is a similar modeling approach as Bhattacharya and Qiao (2007) and Raffin and Seegmuller (2017). The longer the representative agent lives in the elderly period, the more utility she derives from elderly consumption. This specification allows us to focus on the agent's tradeoff between private healthcare expenditure and savings, such that we can still obtain phase diagrams similar to Figure 1 and Figure 2. All else equal, increasing (decreasing) savings raises (lowers) elderly consumption, but decreases (increases) private healthcare expenditure such that the timespan when the agent can enjoy the elderly consumption is shortened (prolonged). The representative agent's lifetime utility is

$$U_t = \phi(x(e_t))u(d_{t+1}), \quad (\text{D.2})$$

where  $u(d_{t+1})$  is assumed to be in the CRRA form, i.e.,  $d_{t+1}u'(d_{t+1})/u(d_{t+1}) = 1 - \sigma$ , and  $\sigma$  is the coefficient of relative risk aversion.

The representative agent faces two budget constraints:

$$w_t = e_t + s_t, \quad (\text{D.3})$$

$$r_{t+1}s_t = d_{t+1}. \quad (\text{D.4})$$

The representative agent chooses private healthcare expenditure  $e_t$ , savings  $s_t$ , and elderly consumption  $d_{t+1}$  to maximize her lifetime utility (D.2) subject to her young budget constraint (D.3), elderly budget constraint (D.4), health status (D.1), and longevity function (9).

Substituting  $r_{t+1}s_t = d_{t+1}$  into the objective function simplifies the problem as

$$\max_{e_t, s_t} U_t = \phi(x(e_t))u(r_{t+1}s_t)$$

$$\text{s.t. } w_t = e_t + s_t.$$

The associated Lagrange is

$$L = \phi(x(e_t))u(r_{t+1}s_t) + \psi_t [w_t - e_t - s_t],$$

where  $\psi_t$  is the Lagrange multiplier of the young-age budget constraint.

The first-order conditions are

$$\phi(x(e_t))u'(d_{t+1})r_{t+1} - \psi_t = 0, \quad (\text{D.5})$$

$$\phi'(x(e_t))x'(e_t)u(d_{t+1}) - \psi_t = 0, \quad (\text{D.6})$$

$$w_t - e_t - s_t = 0. \quad (\text{D.7})$$

Substituting (D.5) into (D.6) to eliminate  $\psi_t$  and multiplying both sides by  $s_t$  yield

$$\phi(x(e_t))u'(d_{t+1})d_{t+1} = s_t\phi'(x(e_t))x'(e_t)u(d_{t+1}), \quad (\text{D.8})$$

which states that the representative agent adjusts her young budget such that the marginal benefit of savings through its effect on elderly consumption is balanced by the marginal benefit of private healthcare expenditure through its effect on longevity. Rearranging (D.8) gives

$$\frac{s_t}{e_t} \left[ \frac{x(e_t)\phi'(x(e_t))}{\phi(x(e_t))} \right] \left[ \frac{e_t x'(e_t)}{x(e_t)} \right] = \frac{d_{t+1}u'(d_{t+1})}{u(d_{t+1})}. \quad (\text{D.9})$$

Because  $x(e_t)\phi'(x(e_t))/\phi(x(e_t)) = 1/(1+x(e_t))$  by Assumption (ii)  $\underline{\lambda} = 0$  for simplicity,

$e_t x'(e_t)/x(e_t) = \mu$ , and  $d_{t+1}u'(d_{t+1})/u(d_{t+1}) = 1 - \sigma$ , equation (D.9) becomes

$$s_t = \frac{1-\sigma}{\mu} e_t [1+x(e_t)]. \quad (\text{D.10})$$

Substituting (D.3) into (D.10) to eliminate  $e_t$  gives the function that determines the representative agent's savings:

$$s_t = \frac{1-\sigma}{\mu} (w_t - s_t) [1+x(w_t - s_t)]. \quad (\text{D.11})$$

Note that in (D.11) there is no closed-form solution for  $s_t$ , but it is straightforward to verify that the agent's savings increase in her wage rate.

### D.1 Robustness under Environmental Regulation with Pollution Permits

Suppose the government implements environmental regulation with pollution permits. In equilibrium, the representative agent's savings  $s_t$  become next period's physical capital  $k_{t+1}$ . Substituting (1), (3), and (D.1) into (D.11) gives the nonlinear difference equation for capital

$$k_{t+1} = \frac{1-\sigma}{\mu} \left[ \beta B \bar{p}^{1-\alpha-\beta} k_t^\alpha - k_{t+1} \right] \left[ 1 + \frac{\left( \beta B \bar{p}^{1-\alpha-\beta} k_t^\alpha - k_{t+1} \right)^\mu \left( B \bar{p}^{1-\alpha-\beta} k_t^\alpha \right)^{1-\mu}}{\eta z_t} \right]. \quad (\text{D.12})$$

From equation (D.12), we define the  $kk_{alternative}^{pp}$  locus where physical capital is in steady state:

$$\frac{1-\sigma}{\mu} \left[ \beta B \bar{p}^{1-\alpha-\beta} k_t^\alpha - k_t \right] \left[ 1 + \frac{\left( \beta B \bar{p}^{1-\alpha-\beta} k_t^\alpha - k_t \right)^\mu \left( B \bar{p}^{1-\alpha-\beta} k_t^\alpha \right)^{1-\mu}}{\eta z_t} \right] - k_t = 0. \quad (\text{D.13})$$

On the environmental side, substituting (5) into (8) and setting  $\bar{p}_t = \bar{p}$  gives

$$z_{t+1} = (1-\theta)z_t - \gamma(1-\alpha-\beta)B\bar{p}^{1-\alpha-\beta}k_t^\alpha + \bar{p}. \quad (\text{D.14})$$

From equation (D.14), we define the  $zz_{alternative}^{pp}$  locus where the stock of pollution is in steady state:

$$-\theta z_t - \gamma(1-\alpha-\beta)B\bar{p}^{1-\alpha-\beta}k_t^\alpha + \bar{p} = 0. \quad (\text{D.15})$$

The capital-environment dynamics under environmental regulation with pollution permits are determined jointly by the difference equations (D.12) and (D.14).

Multiple equilibria also emerge in this alternative model with private healthcare expenditure. The equilibrium featuring low capital and a high stock of pollution is an EPT since all combinations of capital and stock of pollution in its vicinity will gravitate toward it. The other equilibrium exhibits saddle stability. The benchmark parameters are given in Table 1 from the main paper, plus the elasticity of health status with respect to private healthcare expenditure  $\mu = 0.5$ , the coefficient of relative risk aversion in elderly consumption function  $\sigma = 0.9$ , and the number of pollution permits  $\bar{p} = 6.25$ . Using these parameters, we find that the eigenvalues associated with the EPT are 0.67 and -0.07, which lie within the  $[-1,1]$  range and imply that the EPT is stable. The eigenvalues associated with the desirable equilibrium are 1.58 and -0.91. Since one is greater than 1, while the other lies within the  $[-1,1]$  range, the desirable equilibrium exhibits saddle stability. Further, the transition paths in the vicinity of the equilibria in Figure D1 confirm the dynamic properties around the equilibria. In this alternative model with private healthcare expenditure, our main results do not qualitatively change, which demonstrates the robustness of our results that environmental regulation with pollution permits might give rise to an EPT.

## **D.2 Robustness under Environmental Regulation with Green Taxes**

Now suppose the government implements environmental regulation with green taxes. Again, in equilibrium, the representative agent's savings become next period's capital, i.e.,  $s_t = k_{t+1}$ .

Substituting (1), (3), (4), and (D.1) into (D.11) yields

$$k_{t+1} = \frac{1-\sigma}{\mu} \left[ \beta \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} - k_{t+1} \right] \times \left[ 1 + \frac{\left( \beta \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} - k_{t+1} \right)^\mu \left( \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} \right)^{1-\mu}}{\eta z_t} \right]. \quad (\text{D.16})$$

From equation (D.16), we define the  $kk_{alternative}^{gt}$  locus where physical capital is in steady state:

$$\frac{1-\sigma}{\mu} \left[ \beta \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} - k_t \right] \times \left[ 1 + \frac{\left( \beta \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} - k_t \right)^\mu \left( \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1-\alpha-\beta}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} \right)^{1-\mu}}{\eta z_t} \right] - k_t = 0. \quad (\text{D.17})$$

Substituting (6) into (8) gives

$$z_{t+1} = (1-\theta)z_t + (1-\gamma q) \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}}. \quad (\text{D.18})$$

From equation (D.18), we define the  $zz_{alternative}^{gt}$  locus where the stock of pollution is in steady state:

$$-\theta z_t + (1-\gamma q) \left( \frac{1-\alpha-\beta}{q} \right)^{\frac{1}{\alpha+\beta}} B^{\frac{1}{\alpha+\beta}} k_t^{\frac{\alpha}{\alpha+\beta}} = 0. \quad (\text{D.19})$$

The capital-environment dynamics under environmental regulation with green taxes are determined jointly by difference equations (D.16) and (D.18). All the parameters used here are the same as those used under pollution permits, except for the green-tax rate  $q = 0.045$ . The eigenvalues associated with the equilibrium are  $0.33 \pm 0.25i$ , indicating that the equilibrium is spirally stable. The simulations shown in Figure D2 confirm that the equilibrium under a green-

tax policy is spirally stable. Again, the dynamics under the green-tax system do not qualitatively change in the alternative model with private healthcare expenditure.

## Appendix E. Comparative Statics around the Steady States

In Appendix E, we provide analytical results of the long-run impacts from the changes in both policy and technical parameters under pollution permits and under green taxes in our basic model.

### Proof #1. Comparative Statics under Environmental Regulation with Pollution Permits

Equations (B.1) and (B.2) evaluated at the steady states are

$$\Phi(y^{pp}(k, \bar{p}), z, \eta)w^{pp}(k, \bar{p}) - k = 0, \quad (\text{E.1})$$

$$-\theta z - \gamma a^{pp}(k, \bar{p}) + \bar{p} = 0. \quad (\text{E.2})$$

Applying the Implicit Function Theorem to (E.1) and (E.2), and simplifying yields

$$\underbrace{\begin{bmatrix} \alpha(E^{pp} + 1) - 1 & -E^{pp} \frac{k}{z} \\ -\alpha \frac{\gamma a^{pp}}{k} & -\theta \end{bmatrix}}_{D^{pp}} \begin{bmatrix} \frac{\partial k}{\partial \bar{p}} & \frac{\partial k}{\partial \gamma} & \frac{\partial k}{\partial \eta} & \frac{\partial k}{\partial \theta} \\ \frac{\partial z}{\partial \bar{p}} & \frac{\partial z}{\partial \gamma} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \theta} \end{bmatrix} = \begin{bmatrix} -(1 - \alpha - \beta) \frac{k}{\bar{p}} (E^{pp} + 1) & 0 & E^{pp} \frac{k}{\eta} & 0 \\ (1 - \alpha - \beta) \frac{\gamma a^{pp}}{\bar{p}} - 1 & a^{pp} & 0 & z \end{bmatrix},$$

where the determinant of matrix  $D^{pp}$  is

$$|D^{pp}| = -\theta \left[ \alpha(E^{pp} + 1) - 1 \right] - \alpha \left( \frac{\gamma a^{pp}}{z} \right) E^{pp} = \alpha \theta \left( \frac{1 - \alpha}{\alpha} - \frac{\bar{p}}{\theta z} E^{pp} \right).$$

Notice that the sign of  $|D^{pp}|$  depends on the slopes of the  $kk^{pp}$  locus and the  $zz^{pp}$  locus at the steady state by (B.12) and (B.14).

By Cramer's Rule, we get the effects of a change in pollution permits on steady-state capital and on the steady-state stock of pollution:

$$\frac{\partial k}{\partial \bar{p}} = \frac{\theta(\alpha + \beta) \frac{k}{\bar{p}} \left( \frac{1-\alpha-\beta}{\alpha+\beta} - \frac{\bar{p}}{\theta z} E^{pp} \right)}{|D^{pp}|}, \quad (\text{E.3})$$

$$\frac{\partial z}{\partial \bar{p}} = \frac{\theta z \overbrace{\left( \beta - \alpha E^{pp} \right) \frac{\bar{p}}{\theta z} + (1-\alpha-\beta)}^{(+)}}{|D^{pp}|}. \quad (\text{E.4})$$

At the steady state  $(k^l, z^h)$ ,  $\frac{1-\alpha}{\alpha} - \frac{\bar{p}}{\theta z} E^{pp} > 0$  and  $|D^{pp}| > 0$ . Also notice that  $\frac{1-\alpha-\beta}{\alpha+\beta} < \frac{1-\alpha}{\alpha}$ .

An increase in  $\bar{p}$  may increase or decrease  $k^l$  depending on the relative magnitudes of  $\frac{1-\alpha-\beta}{\alpha+\beta}$  and  $\frac{\bar{p}}{\theta z} E^{pp}$  evaluated at  $(k^l, z^h)$ , but unambiguously increases  $z^h$ . In contrast, at steady state  $(k^h, z^l)$ ,  $\frac{1-\alpha}{\alpha} - \frac{\bar{p}}{\theta z} E^{pp} < 0$  and  $|D^{pp}| < 0$ . An increase in  $\bar{p}$  unambiguously increases  $k^h$  and decreases  $z^l$ .

The following are the effects of changes in parameters  $\gamma$ ,  $\eta$ , and  $\theta$  on the steady states:

$$\frac{\partial k}{\partial \gamma} = \frac{k a^{pp} E^{pp}}{z |D^{pp}|}, \quad (\text{E.5})$$

$$\frac{\partial z}{\partial \gamma} = \frac{\alpha a^{pp} \overbrace{\left( E^{pp} - \frac{1-\alpha}{\alpha} \right)}^{(-)}}{|D^{pp}|}, \quad (\text{E.6})$$

$$\frac{\partial k}{\partial \eta} = -\frac{\theta k E^{pp}}{\eta |D^{pp}|}, \quad (\text{E.7})$$

$$\frac{\partial z}{\partial \eta} = \frac{\alpha \gamma a^{pp} E^{pp}}{\eta |D^{pp}|}, \quad (\text{E.8})$$

$$\frac{\partial k}{\partial \theta} = \frac{k E^{pp}}{|D^{pp}|}, \quad (\text{E.9})$$

$$\frac{\partial z}{\partial \theta} = \frac{\alpha z \overbrace{\left( E^{pp} - \frac{1-\alpha}{\alpha} \right)}^{(-)}}{|D^{pp}|}. \quad (\text{E.10})$$

At the steady state  $(k^l, z^h)$ ,  $|D^{pp}| > 0$ . Increases in  $\gamma$  and  $\theta$  unambiguously increase  $k^l$  and decrease  $z^h$ . Based on the relevant expressions, both longevity and period welfare in the steady state also increase as these parameters increase. An increase in  $\eta$  unambiguously decreases  $k^l$  and increases  $z^h$ , and longevity and period welfare decrease in  $\eta$ . In contrast, at the steady state  $(k^h, z^l)$ ,  $|D^{pp}| < 0$ . Increases in  $\gamma$  and  $\theta$  unambiguously decrease  $k^h$  and increase  $z^l$ . So both longevity and period welfare in the steady state decrease as these parameters increase. An increase in  $\eta$  has the opposite effects on  $k^h$ ,  $z^l$ , longevity, and period welfare.

### Proof #2. Comparative Statics under Environmental Regulation with Green Taxes

Equations (C.1) and (C.2) evaluated at the steady state are

$$\Phi(y^{gt}(k, q), z, \eta)w^{gt}(k, q) - k = 0, \quad (\text{E.11})$$

$$-\theta z + (1 - \gamma q)p^{gt}(k, q) = 0. \quad (\text{E.12})$$

Applying the Implicit Function Theorem to (E.11) and (E.12) yields

$$\underbrace{\begin{bmatrix} \frac{\alpha}{\alpha+\beta}(E^{gt}+1)-1 & -E^{gt} \frac{k}{z} \\ \frac{\alpha}{\alpha+\beta} \frac{\theta z}{k} & -\theta \end{bmatrix}}_{D^{gt}} \begin{bmatrix} \frac{\partial k}{\partial q} & \frac{\partial k}{\partial \gamma} & \frac{\partial k}{\partial \eta} & \frac{\partial k}{\partial \theta} \\ \frac{\partial z}{\partial q} & \frac{\partial z}{\partial \gamma} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{1-\alpha-\beta}{\alpha+\beta} \frac{k}{q}(E^{gt}+1) & 0 & E^{gt} \frac{k}{\eta} & 0 \\ \frac{\theta z}{q} \left( \frac{1}{1-\gamma q} + \frac{1-\alpha-\beta}{\alpha+\beta} \right) & qp^{gt} & 0 & z \end{bmatrix},$$

where the determinant of matrix  $D^{gt}$  is

$$|D^{gt}| = -\theta \left[ \frac{\alpha}{\alpha+\beta}(E^{gt}+1)-1 \right] + \theta \frac{\alpha}{\alpha+\beta} E^{gt} = \frac{\theta\beta}{\alpha+\beta} > 0.$$

By Cramer's Rule, we get the effects of a change in the green-tax rate on the steady-state capital and on the steady-state stock of pollution:

$$\frac{\partial k}{\partial q} = \frac{\theta k}{q(1-\gamma q)} \frac{\left[ E^{gt} - \frac{1-\alpha-\beta}{\alpha+\beta} (1-\gamma q) \right]}{|D^{gt}|}, \quad (\text{E.13})$$

$$\frac{\partial z}{\partial q} = \frac{\alpha}{\alpha+\beta} \frac{\theta z}{(1-\gamma q)q} \frac{\overbrace{\left[ E^{gt} - \frac{\beta}{\alpha} - \frac{1-\alpha-\beta}{\alpha} (1-\gamma q) \right]}^{(-)}}{|D^{gt}|} < 0. \quad (\text{E.14})$$

An increase in  $q$  has an indeterminate effect on the steady-state capital depending on the sign of  $E^{gt} - \frac{1-\alpha-\beta}{\alpha+\beta} (1-\gamma q)$ , but unambiguously decreases the steady-state stock of pollution.

The effects of changes in parameters  $\gamma$ ,  $\eta$ , and  $\theta$  on the steady state are as follows:

$$\frac{\partial k}{\partial \gamma} = \frac{k}{z} \frac{qp^{gt} E^{gt}}{|D^{gt}|} > 0, \quad (\text{E.15})$$

$$\frac{\partial z}{\partial \gamma} = \frac{qp^{gt}}{|D^{gt}|} \frac{\overbrace{\left[ \frac{\alpha}{\alpha+\beta} (E^{gt} + 1) - 1 \right]}^{(-)}}{|D^{gt}|} < 0, \quad (\text{E.16})$$

$$\frac{\partial k}{\partial \eta} = -\frac{\theta k}{\eta} \frac{E^{gt}}{|D^{gt}|} < 0, \quad (\text{E.17})$$

$$\frac{\partial z}{\partial \eta} = -\frac{\alpha}{\alpha+\beta} \frac{\theta z}{\eta} \frac{E^{gt}}{|D^{gt}|} < 0, \quad (\text{E.18})$$

$$\frac{\partial k}{\partial \theta} = \frac{kE^{gt}}{|D^{gt}|} > 0, \quad (\text{E.19})$$

$$\frac{\partial z}{\partial \theta} = \frac{z \overbrace{\left[ \frac{\alpha}{\alpha+\beta} (E^{gt} + 1) - 1 \right]}^{(-)}}{|D^{gt}|} < 0, \quad (\text{E.20})$$

The signs of these terms are unambiguous. Increases in  $\gamma$  and  $\theta$  increase the steady-state capital and decrease the steady-state stock of pollution. So longevity and period welfare in the steady state increase in  $\gamma$  and  $\theta$ . However, an increase in  $\eta$  decreases both the steady-state capital and the steady-state stock of pollution.

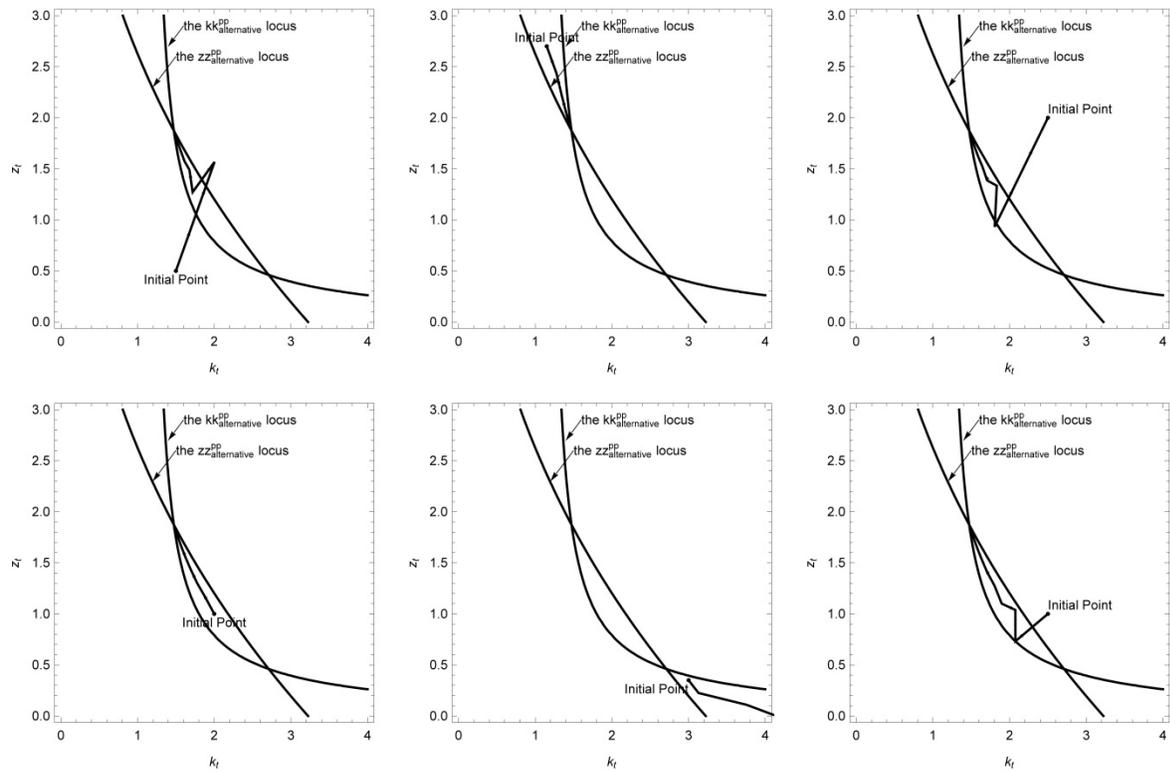


Figure D1. Dynamics under Pollution Permits with Private Healthcare Expenditure

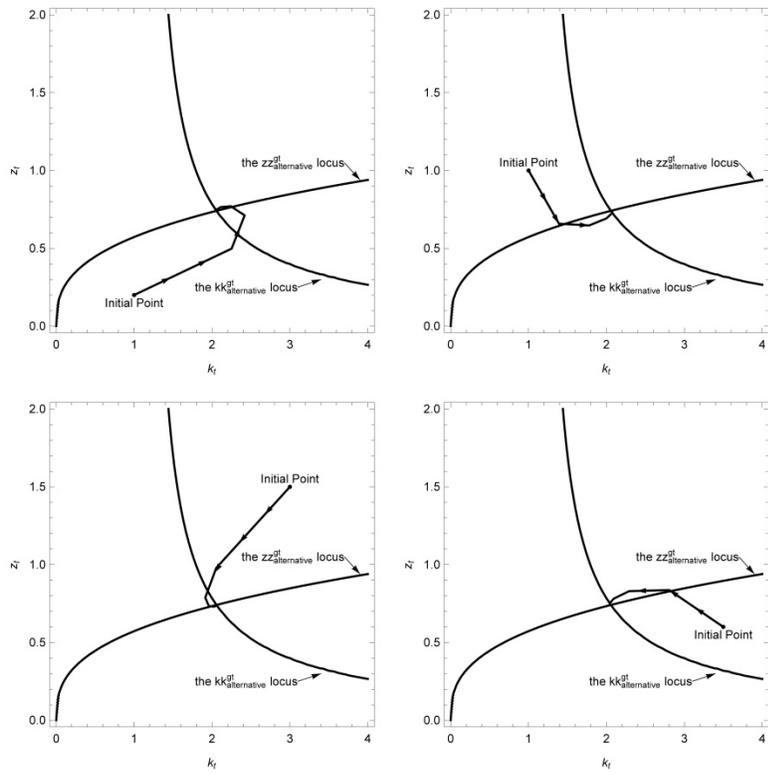


Figure D2. Dynamics under Green Taxes with Private Healthcare Expenditure

## References

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