# Pollution Permits, Green Taxes, and the Environmental Poverty Trap

**(Supplementary Document)**

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## Appendix A. The Social Planner's Problem

The production function in per-capita terms is

$$y_t = Bk_t^{\alpha} p_t^{1-\alpha-\beta}, \tag{A.1}$$

and the lifetime utility of the agent born at the beginning of period $t$ is

$$U_t = U(c_t, d_{t+1}, p_t, k_t, z_t) = \ln c_t + \frac{y(k_t, p_t)}{\eta z_t + y(k_t, p_t)} \ln d_{t+1}. \tag{A.2}$$

In period $t$ in per-capita terms, total output $y_t$ becomes young consumption $c_t$, elderly consumption $d_t$, environmental maintenance $a_t$, and next period's physical capital $k_{t+1}$. The resource constraint is

$$y_t - c_t - d_t - a_t - k_{t+1} = 0. \tag{A.3}$$

The dynamics for the stock of pollution is

$$z_{t+1} - (1-\theta)z_t + \gamma a_t - p_t = 0. \tag{A.4}$$

With $\delta \in (0,1)$ as the discount factor, the social planner's problem is to maximize the discounted sum of agents' utility

$$\sum_{t=0}^{+\infty} \delta^t U_t \,, \tag{A.5}$$

subject to (A.3) and (A.4) by choosing young consumption $c_t$, elderly consumption $d_t$, environmental maintenance $a_t$, pollution flow $p_t$, as well as physical capital $k_t$ and the stock of pollution $z_t$. The Lagrangian associated with the social planner's problem is

$$\ell = \sum_{t=0}^{+\infty} \delta^t \left[ U_t + \omega_{t+1}\left( y_t - c_t - d_t - a_t - k_{t+1} \right) + \nu_{t+1}\left( z_{t+1} - (1-\theta) z_t + \gamma a_t - p_t \right) \right], \tag{A.6}$$

where $\omega_{t+1}$ and $\nu_{t+1}$ are Lagrangian multipliers of resource constraint and the stock of pollution dynamics.

Taking derivatives of $\ell$ in (A.6) with respect to $c_t$, $d_t$, $a_t$, $p_t$, $k_t$, and $z_t$, we have

$$c_t: \quad \frac{\partial U_t}{\partial c_t} = \omega_{t+1}, \tag{A.7}$$

$$d_t: \quad \frac{\partial U_{t-1}}{\partial d_t} = \delta \omega_{t+1}, \tag{A.8}$$

$$a_t: \quad \gamma \nu_{t+1} = \omega_{t+1}, \tag{A.9}$$

$$p_t: \quad \frac{\partial U_t}{\partial p_t} + \omega_{t+1} \frac{\partial y_t}{\partial p_t} = \nu_{t+1}, \tag{A.10}$$

$$k_t: \quad \delta \left[ \frac{\partial U_t}{\partial k_t} + \omega_{t+1} \frac{\partial y_t}{\partial k_t} \right] = \omega_t, \tag{A.11}$$

$$z_t: \quad \delta \left( (1-\theta)\nu_{t+1} - \frac{\partial U_t}{\partial z_t} \right) = \nu_t. \tag{A.12}$$

Substituting (A.7) into (A.8) to eliminate $\omega_{t+1}$ and writing at period $t+1$ gives the tradeoff between generations:

$$\begin{aligned} \frac{\partial U_t}{\partial d_{t+1}} &= \delta \frac{\partial U_{t+1}}{\partial c_{t+1}}, \\ \frac{y_t}{\eta z_t + y_t} \frac{1}{d_{t+1}} &= \delta \frac{1}{c_{t+1}}. \end{aligned} \tag{A.13}$$

From (A.7), (A.9), and (A.10), we have the tradeoff between the flow of pollution and young consumption:

$$\begin{aligned} \frac{\partial U_t}{\partial p_t} &= \left( \frac{1}{\gamma} - \frac{\partial y_t}{\partial p_t} \right) \frac{\partial U_t}{\partial c_t}, \\ \frac{\partial y_t}{\partial p_t} \frac{\eta z_t}{(\eta z_t + y_t)^2} \ln d_{t+1} &= \left( \frac{1}{\gamma} - \frac{\partial y_t}{\partial p_t} \right) \frac{1}{c_t}. \end{aligned} \tag{A.14}$$

Rewriting (A.11) at period $t+1$, substituting (A.7) in to eliminate $\omega_{t+1}$, and substituting (A.8) in to eliminate $\omega_{t+2}$ gives the tradeoff between consumptions over an agent's life cycle:

$$\begin{aligned} \delta \frac{\partial U_{t+1}}{\partial k_{t+1}} + \frac{\partial y_{t+1}}{\partial k_{t+1}} \frac{\partial U_t}{\partial d_{t+1}} &= \frac{\partial U_t}{\partial c_t}, \\ \frac{\partial y_{t+1}}{\partial k_{t+1}} \left[ \delta \frac{\eta z_{t+1}}{(\eta z_{t+1} + y_{t+1})^2} \ln d_{t+2} + \frac{y_t}{\eta z_t + y_t} \frac{1}{d_{t+1}} \right] &= \frac{1}{c_t}. \end{aligned} \tag{A.15}$$

Rewriting (A.12) at period $t+1$ and substituting (A.7) and (A.9) in gives the tradeoff between consumption and the stock of pollution:

$$\begin{aligned} \delta \left[ (1-\theta) \frac{\partial U_{t+1}}{\partial c_{t+1}} - \gamma \frac{\partial U_{t+1}}{\partial z_{t+1}} \right] &= \frac{\partial U_t}{\partial c_t}, \\ \delta \left[ (1-\theta) \frac{1}{c_{t+1}} + \frac{\gamma \eta y_{t+1}}{(\eta z_{t+1} + y_{t+1})^2} \ln d_{t+2} \right] &= \frac{1}{c_t}. \end{aligned} \tag{A.16}$$

Using (A.3) and (A.4) to eliminate $a_t$ gives

$$z_{t+1} - (1-\theta)z_t + \gamma(y_t - c_t - d_t - k_{t+1}) - p_t = 0 . \qquad \text{(A.17)}$$

Next, we find the steady-state solution to the social planner's problem with 5 equations in steady state, (A.13)-(A.17), and 5 unknown steady-state values, $c, d, p, k,$ and $z$. Substituting the entire expressions of (A.13) and (A.14) into (A.15) gives $k$ as a function of $p$:

$$k = \frac{\delta}{\gamma}\frac{\alpha}{1-\alpha-\beta}\, p .$$

Substituting the expression of (A.14) into (A.16) gives $z$ as a function of $p$:

$$z = \frac{\frac{\delta}{1-\alpha-\beta}\, p - \gamma\delta B\left(\frac{\delta}{\gamma}\frac{\alpha}{1-\alpha-\beta}\right)^{\alpha} p^{1-\beta}}{1-\delta(1-\theta)} .$$

Substituting $k$ and $z$ as functions of $p$ into (A.13) and (A.17) gives two equations with $p$, $c$, and $d$. So we can express $c$ and $d$ as functions of $p$. Substituting $c, d, k,$ and $z$ as functions of $p$ into (A.14) gives an equation with only $p$, which leads to the steady-state value for $p$ and further the steady-state values for $c, d, a, k,$ and $z$.

## Appendix B. Proofs under Environmental Regulation with Pollution Permits

In Appendix B, we prove Proposition 1 that summarizes the conditions for the emergence of multiple equilibria under pollution permits. Then we focus on the case where multiple equilibria emerge and the conditions under which multiple (dual) equilibria are satisfied. To show the local dynamic properties surrounding the two nontrivial steady states, we divide the subsequent analysis into four parts. First, we analytically prove the slopes of the $kk^{pp}$ locus and the $zz^{pp}$ locus. Second, we figure out the relative positions of the $kk^{pp}$ locus and the $zz^{pp}$ locus. Third, we prove Proposition 2 based on the slopes of the $kk^{pp}$ locus and the $zz^{pp}$ locus. Fourth, we check the global curvature of the $kk^{pp}$ locus to eliminate the third possible steady state and corroborate Proposition 1.

<u>Proof #1</u>. Proposition 1.

Under the Assumptions (i) $0<\alpha<\beta<1$ and (ii) $\underline{\lambda}=0$ and $\overline{\lambda}=1$, substituting (23) into (20) to eliminate $z_t$ and rearranging gives

$$\frac{1}{\eta}\beta B^2\overline{p}^{2(1-\alpha-\beta)}=\frac{\overline{p}}{\theta}k_t^{1-2\alpha}+\left[\frac{2}{\eta}-\frac{\gamma}{\theta}(1-\alpha-\beta)\right]B\overline{p}^{1-\alpha-\beta}k_t^{1-\alpha}\equiv T(k_t).$$

For $T(k_t)$, $T(0)=0$ and $T'(k_t)=(1-2\alpha)\frac{\overline{p}}{\theta}k_t^{-2\alpha}+(1-\alpha)\left[\frac{2}{\eta}-\frac{\gamma}{\theta}(1-\alpha-\beta)\right]B\overline{p}^{1-\alpha-\beta}k_t^{-\alpha}$. First, when $\frac{2}{\eta}-\frac{\gamma}{\theta}(1-\alpha-\beta)\geq 0 \Leftrightarrow \gamma\leq\frac{2}{1-\alpha-\beta}\frac{\theta}{\eta}$, $T'(k_t)>0$ for $k_t>0$, and only one solution for $k_t$ exists. In the second case, $\frac{2}{\eta}-\frac{\gamma}{\theta}(1-\alpha-\beta)<0 \Leftrightarrow \gamma>\frac{2}{1-\alpha-\beta}\frac{\theta}{\eta}$. If $0<k_t<\left[\frac{1-2\alpha}{1-\alpha}\frac{\overline{p}^{\alpha+\beta}}{\left[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}\right]B}\right]^{\frac{1}{\alpha}}$, $T'(k_t)>0$; if $k_t=\left[\frac{1-2\alpha}{1-\alpha}\frac{\overline{p}^{\alpha+\beta}}{\left[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}\right]B}\right]^{\frac{1}{\alpha}}$, $T'(k_t)=0$; if $k_t>\left[\frac{1-2\alpha}{1-\alpha}\frac{\overline{p}^{\alpha+\beta}}{\left[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}\right]B}\right]^{\frac{1}{\alpha}}$, $T'(k_t)<0$. So the function $T(k_t)$ first rises, reaches its peak at $k_t=\left[\frac{1-2\alpha}{1-\alpha}\frac{\overline{p}^{\alpha+\beta}}{\left[\gamma(1-\alpha-\beta)-2\frac{\theta}{\eta}\right]B}\right]^{\frac{1}{\alpha}}\equiv k^*$, and then falls. So the emergence of multiple equilibria also requires that $\frac{1}{\eta}\beta B^2\overline{p}^{2(1-\alpha-\beta)}$ must be lower than the peak of $T(k_t)$, i.e., $T(k^*)$.

Next, we focus on the case where multiple equilibria emerge. We will show that without introducing additional assumptions other than Assumptions (i) and (ii), the following facts can be established.

<u>Proof #2</u>. The $kk^{pp}$ locus and the $zz^{pp}$ locus slope down in $(k_t, z_t)$ space.

Start from equations (20) and (23):

$$\Phi(y^{pp}(k_t, \overline{p}), z_t) w^{pp}(k_t, \overline{p}) - k_t = 0 , \tag{B.1}$$

$$-\theta z_t - \gamma a^{pp}(k_t, \overline{p}) + \overline{p} = 0 . \tag{B.2}$$

Now we check the slope of the $kk^{pp}$ locus. Totally differentiating (B.1) and rearranging gives

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } kk^{pp} \text{ locus}} = -\frac{\frac{\partial \Phi_{t+1}}{\partial y_t^{pp}} \frac{\partial y_t^{pp}}{\partial k_t} w_t^{pp} + \Phi_{t+1} \frac{\partial w_t^{pp}}{\partial k_t} - 1}{\frac{\partial \Phi_{t+1}}{\partial z_t} w_t^{pp}} . \tag{B.3}$$

Substituting (B.1) into (B.3) and rewriting gives

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } kk^{pp} \text{ locus}} = -\frac{\left(\frac{\partial \Phi_{t+1}}{\partial y_t^{pp}} \frac{y_t^{pp}}{\Phi_{t+1}}\right)\left(\frac{\partial y_t^{pp}}{\partial k_t} \frac{k_t}{y_t^{pp}}\right) + \left(\frac{\partial w_t^{pp}}{\partial k_t} \frac{k_t}{w_t^{pp}}\right) - 1}{\left(\frac{\partial \Phi_{t+1}}{\partial z_t} \frac{z_t}{\Phi_{t+1}}\right) \frac{k_t}{z_t}} . \tag{B.4}$$

Define $E_{\Phi_{t+1}, y_t^{pp}} = \frac{\partial \Phi_{t+1}}{\partial y_t^{pp}} \frac{y_t^{pp}}{\Phi_{t+1}}$ as the elasticity of the propensity to save $\Phi_{t+1}$ with respect to income per capita $y_t^{pp}$, and $E_{\Phi_{t+1}, z_t} = \frac{\partial \Phi_{t+1}}{\partial z_t} \frac{z_t}{\Phi_{t+1}}$ as the elasticity of the propensity to save $\Phi_{t+1}$ with respect to the stock of pollution $z_t$. The elasticities are contained in the ranges $E_{\Phi_{t+1}, y_t^{pp}} \in [0,1]$ and $E_{\Phi_{t+1}, z_t} \in [-1, 0]$. The ranges for both of these elasticities can be verified by substituting (10) into $\Phi_{t+1}$. It also can be verified that $E_t^{pp} = E_{\Phi_{t+1}, y_t^{pp}} = -E_{\Phi_{t+1}, z_t}$, an important fact that will be used later.

Lastly, $E_{w_t^{pp},k_t} = \frac{\partial w_t^{pp}}{\partial k_t}\frac{k_t}{w_t^{pp}}$ is the elasticity of wage rate with respect to capital, $E_{y_t^{pp},k_t} = \frac{\partial y_t^{pp}}{\partial k_t}\frac{k_t}{y_t^{pp}}$ is the elasticity of income per capita with respect to capital, and both are equal to capital's share in production $\alpha$. Thus, (B.4) can be rewritten as

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } kk^{pp} \text{ locus}} = \left(\alpha - \frac{1-\alpha}{E_t^{pp}}\right)\frac{z_t}{k_t} \tag{B.5}$$

The sufficient condition for $dz_t/dk_t|_{\text{the } kk^{pp} \text{ locus}} < 0$ is Assumption (i) $0 < \alpha < \beta < 1$, which combined with $1-\alpha-\beta > 0$ implies that $0 < \alpha < 1/2$. So as long as capital's share in production is less than labor's share, the $kk^{pp}$ locus always slopes down in $(k_t, z_t)$ space.

Now we check the slope of the $zz^{pp}$ locus. Totally differentiating (B.2) and using (5), we get

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } zz^{pp} \text{ locus}} = -\frac{\alpha\gamma}{\theta}\frac{a_t^{pp}}{k_t}. \tag{B.6}$$

It is straightforward that $dz_t/dk_t|_{\text{the } zz^{pp} \text{ locus}} < 0$ from (B.6), suggesting that the $zz^{pp}$ locus slopes down in $(k_t, \mathrm{z}_t)$ space.

<u>Proof #3</u>. The relative positions of the $kk^{pp}$ locus and the $zz^{pp}$ locus in $(k_t, \mathrm{z}_t)$ space.

On the $kk^{pp}$ locus, $k_t$ asymptotically converges to $\left(\frac{\underline{\lambda}}{\underline{\lambda}+1}\beta B\overline{p}^{1-\alpha-\beta}\right)^{\frac{1}{1-\alpha}}$ when $z_t \to +\infty$. This fact can be established by using (3), (18), and (20). As $z_t \to +\infty$, the health status $x_t \to 0$. So the propensity to save converges to its lower bound $\frac{\underline{\lambda}}{\underline{\lambda}+1}$, and physical capital converges to $\left(\frac{\underline{\lambda}}{\underline{\lambda}+1}\beta B\overline{p}^{1-\alpha-\beta}\right)^{\frac{1}{1-\alpha}}$. On the $zz^{pp}$ locus, in contrast, when $k_t = 0$, $z_t = \overline{p}/\theta$ by (23). So when $k_t$ is relatively small, the $kk^{pp}$ locus lies above the $zz^{pp}$ locus in $(k_t, z_t)$ space. Combining the fact that both loci monotonically decrease in $k_t$, we conclude that as $k_t$ increase, the $kk^{pp}$ locus must first intersect with the $zz^{pp}$ locus from above, indicating that at the steady state $(k^l, z^h)$, the slope of the $kk^{pp}$ locus must be steeper than that of the $zz^{pp}$ locus. It follows that as $k_t$ increases until the two loci intersect again at $(k^h, z^l)$, the slope of the $kk^{pp}$ locus must be flatter than that of the $zz^{pp}$ locus.

Next, we analytically prove Proposition 2 by showing that the type of (local) transition dynamics is stable around a steady state where the slope of the $kk^{pp}$ locus is steeper than that of the $zz^{pp}$ locus, while a steady state where the slope of the $kk^{pp}$ locus is flatter than that of the $zz^{pp}$ locus can be unstable or exhibits saddle-path stability.[1]

Proof #4. Proposition 2

Rewrite equations (18) and (21) as

$$k_{t+1} = \Phi\left( y^{pp}(k_t), z_t \right) w^{pp}(k_t) , \tag{B.7}$$

$$z_{t+1} = (1-\theta) z_t - \gamma a^{pp}(k_t) + \overline{p} . \tag{B.8}$$

Then, totally differentiate (B.7) and (B.8) around the steady states to produce

[1] Steady state 'C' in Figure 1 from the main paper exhibits saddle-path stability. Any initial levels of capital to the left of the saddle path will lead to a path transitioning into the undesirable steady state 'B'. Initial levels of capital to the right of the saddle path will lead to a path that eventually violates the condition $z \geq 0$. We have ruled out such paths. However, we acknowledge that it may be possible to modify the model such that corner solutions are allowed where $z = 0$, and any excess government revenues are either returned to individuals or spent on other government programs. Despite this possible modification, the main results in this paper, i.e., the emergence of an EPT under pollution permits and the policies required to avoid it, do not change. We leave such a possibility for future research.

$$dk_{t+1} = \left[ \frac{\partial \Phi}{\partial y^{pp}} \frac{\partial y^{pp}}{\partial k} w^{pp} + \Phi \frac{\partial w^{pp}}{\partial k} \right] dk_t + \frac{\partial \Phi}{\partial z} w^{pp} dz_t,$$

$$dz_{t+1} = -\gamma \frac{\partial a^{pp}}{\partial k} dk_t + (1-\theta) dz_t,$$

where the subscripts $t$ of functions and variables are all left out to indicate that the partial derivatives are evaluated at the steady state, either at $(k^l, z^h)$ or at $(k^h, z^l)$. The associated Jacobian matrix is

$$J^{pp} = \begin{pmatrix} \frac{\partial \Phi}{\partial y^{pp}} \frac{\partial y^{pp}}{\partial k} w^{pp} + \Phi \frac{\partial w^{pp}}{\partial k} & \frac{\partial \Phi}{\partial z} w^{pp} \\ -\gamma \frac{\partial a^{pp}}{\partial k} & 1-\theta \end{pmatrix}. \tag{B.9}$$

Next, substituting (B.1) into (B.9) and rearranging gives

$$J^{pp} = \begin{pmatrix} \alpha(E^{pp}+1) & -E^{pp} \frac{k}{z} \\ -\gamma\alpha \frac{a^{pp}}{k} & 1-\theta \end{pmatrix}, \tag{B.10}$$

where $E_{w^{pp},k} = \frac{\partial w^{pp}}{\partial k} \frac{k}{w^{pp}} = \alpha$ is the elasticity of the wage rate with respect to capital,

$E_{y^{pp},k} = \frac{\partial y^{pp}}{\partial k} \frac{k}{y^{pp}} = \alpha$ is the elasticity of the income per capita with respect to capital,

$E_{\Phi,y^{pp}} = \frac{\partial \Phi}{\partial y^{pp}} \frac{y^{pp}}{\Phi} = E^{pp}$ is the elasticity of the propensity to save with respect to income per capita, and $E_{\Phi,z} = \frac{\partial \Phi}{\partial z} \frac{z}{\Phi} = -E^{pp}$ is the elasticity of the propensity to save with respect to the stock of pollution. All the elasticities are evaluated at the steady state.

The trace and determinant of the Jacobian matrix are

$$TrJ^{pp} = \alpha\left(E^{pp}+1\right)+(1-\theta) < 2,$$
$$DeJ^{pp} = (1-\theta)\alpha\left(E^{pp}+1\right)-\alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp} < 1.$$

Because $E^{pp} \in [0,1]$ and $\theta \in (0,1]$, both inequalities must hold when $\alpha \in (0,1/2)$. Further, it is straightforward to verify that the eigenvalues are real and distinct because

$$\left(TrJ^{pp}\right)^2 - 4DeJ^{pp} = \left[\alpha\left(E^{pp}+1\right)+(1-\theta)\right]^2 - 4(1-\theta)\alpha\left(E^{pp}+1\right)+4\alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp}$$
$$= \left[\alpha\left(E^{pp}+1\right)-(1-\theta)\right]^2 + 4\alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp} > 0.$$

The characteristic polynomial is $p(\nu) = \nu^2 - \left(TrJ^{pp}\right)\nu + DeJ^{pp}$, and

$$p(1) = 1 - TrJ^{pp} + DeJ^{pp} = \alpha\theta\left(\frac{1-\alpha}{\alpha} - \frac{\overline{p}}{\theta z}E^{pp}\right),$$
$$p(-1) = 1 + TrJ^{pp} + DeJ^{pp} = \alpha\theta\left[\left(\frac{2}{\theta}-1\right)\frac{1+\alpha}{\alpha} + \left(\frac{2}{\theta} - \frac{\overline{p}}{\theta z}\right)E^{pp}\right],$$

where both expressions make use of (B.2).

For the steady state $(k^l, z^h)$ where the slope of the $kk^{pp}$ locus is steeper than that of the $zz^{pp}$ locus, by (B.5) and (B.6), the condition is

$$\left(\alpha - \frac{1-\alpha}{E^{pp}}\right)\frac{z}{k} < -\frac{\alpha\gamma}{\theta}\frac{a^{pp}}{k}. \tag{B.11}$$

Rearranging (B.11) and using (B.2) gives

$$\frac{1-\alpha}{\alpha} - \frac{\overline{p}}{\theta z}E^{pp} > 0\,. \tag{B.12}$$

From (B.12), it is straightforward that $p(1) > 0$. Because

$$p(-1)=\alpha\theta\left[\left(\frac{2}{\theta}-1\right)\frac{1+\alpha}{\alpha}+\left(\frac{2}{\theta}-\frac{\overline{p}}{\theta z}\right)E^{pp}\right]>\alpha\theta\left(\frac{1-\alpha}{\alpha}-\frac{\overline{p}}{\theta z}E^{pp}\right)=p(1)>0,$$

so $p(-1)>0$. Further, it is already shown that $DeJ^{pp}<1$. We thus conclude that the two eigenvalues associated with the steady state $(k^l,z^h)$ lie in the interval $(-1,1)$, and the steady state $(k^l,z^h)$ is a stable equilibrium.

In contrast, for the steady state $(k^h,z^l)$ where the slope of the $kk^{pp}$ locus is flatter than that of the $zz^{pp}$ locus, by (B.5) and (B.6), the condition is

$$\left(\alpha-\frac{1-\alpha}{E^{pp}}\right)\frac{z}{k}>-\frac{\alpha\gamma}{\theta}\frac{a^{pp}}{k}. \tag{B.13}$$

Rearranging gives

$$\frac{1-\alpha}{\alpha}-\frac{\overline{p}}{\theta z}E^{pp}<0, \tag{B.14}$$

so $p(1)<0$. For $p(-1)$, because $p(-1)>p(1)$, there are two possible cases. If the steady-state stock of pollution is sufficiently large, i.e. $z^l>\overline{p}\Big/\left(\frac{1+\alpha}{\alpha}\frac{2-\theta}{E^{pp}}+2\right)$, where $E^{pp}$ is the elasticity evaluated at the steady state $(k^h,z^l)$, so $p(-1)>0$. One eigenvalue lies between $(-1,1)$, while the other is greater than 1, indicating that the steady state $(k^h,z^l)$ is a saddle equilibrium. However, if the steady-state stock of pollution is sufficiently small, i.e., $z^l<\overline{p}\Big/\left(\frac{1+\alpha}{\alpha}\frac{2-\theta}{E^{pp}}+2\right)$, where $E^{pp}$ is the elasticity evaluated at the steady state $(k^h,z^l)$, so $p(-1)<0$. Since one eigenvalue is greater than 1 and the other is smaller than -1, the steady state $(k^h,z^l)$ is not stable.

<u>Proof #5</u>. The global curvature of the $kk^{pp}$ locus in $(k_t,z_t)$ space.

We now check the curvature of the $kk^{pp}$ locus. The reason for doing this is that if the $kk^{pp}$ locus is concave for some range of $k_t$ while convex for another range, more than two non-trivial steady states may emerge and the desirable steady state may be stable rather than saddle. Note that $E_t^{pp} = E(y^{pp}(k_t, \overline{p}), z_t)$ and $z_t = z(k_t)$. Differentiating (B.5) with respect to $k_t$ gives

$$\left.\frac{d^2 z_t}{dk_t^2}\right|_{\text{the } kk^{pp} \text{ locus}} = \frac{1-\alpha}{(E_t^{pp})^2}\left(\frac{\partial E_t^{pp}}{\partial y_t^{pp}}\frac{\partial y_t^{pp}}{\partial k_t} + \frac{\partial E_t^{pp}}{\partial z_t}\frac{\partial z_t}{\partial k_t}\right)\frac{z_t}{k_t} + \left(\alpha - \frac{1-\alpha}{E_t^{pp}}\right)\left(\frac{\partial z_t}{\partial k_t}\frac{1}{k_t} - \frac{z_t}{k_t^2}\right). \qquad \text{(B.15)}$$

Substituting (B.5) into (B.15) to eliminate $\partial z_t / \partial k_t$ and rewriting gives

$$\begin{aligned}\left.\frac{d^2 z_t}{dk_t^2}\right|_{\text{the } kk^{pp} \text{ locus}} &= \frac{1-\alpha}{(E_t^{pp})^2}\left[\left(\frac{\partial E_t^{pp}}{\partial y_t^{pp}}\frac{y_t^{pp}}{E_t^{pp}}\right)\left(\frac{\partial y_t^{pp}}{\partial k_t}\frac{k_t}{y_t^{pp}}\right)\frac{E_t^{pp}}{k_t} + \left(\frac{\partial E_t^{pp}}{\partial z_t}\frac{z_t}{E_t^{pp}}\right)\left(\alpha - \frac{1-\alpha}{E_t^{pp}}\right)\frac{E_t^{pp}}{k_t}\right]\frac{z_t}{k_t} \\ &+ \left(\alpha - \frac{1-\alpha}{E_t^{pp}}\right)\left(\alpha - \frac{1-\alpha}{E_t^{pp}} - 1\right)\frac{z_t}{k_t^2}.\end{aligned} \qquad \text{(B.16)}$$

Intensive math shows that $\tilde{E}_t^{pp} = \frac{\partial E_t^{pp}}{\partial y_t^{pp}}\frac{y_t^{pp}}{E_t^{pp}} = -\frac{\partial E_t^{pp}}{\partial z_t}\frac{z_t}{E_t^{pp}}$, where $\tilde{E}_t^{pp}$ is the elasticity's elasticity. Also note that $\frac{\partial y_t^{pp}}{\partial k_t}\frac{k_t}{y_t^{pp}} = \alpha$. Substituting these expressions into (B.16) and rearranging gives

$$\left.\frac{d^2 z_t}{dk_t^2}\right|_{\text{the } kk^{pp} \text{ locus}} = -\frac{(1-\alpha)z_t}{(E_t^{pp}k_t)^2}\left[\alpha(E_t^{pp})^2 - (1-2\alpha)E_t^{pp} - (1-\alpha)(\tilde{E}_t^{pp}+1)\right]. \qquad \text{(B.17)}$$

Under Assumption (ii), the lower bound of longevity is zero, i.e., $\underline{\lambda} = 0$. The relationship between the elasticity $E_t^{pp}$ and the elasticity's elasticity $\tilde{E}_t^{pp}$ collapses to $\tilde{E}_t^{pp} + 1 = E_t^{pp}$ and (B.17) collapses to $\left.\frac{d^2 z_t}{dk_t^2}\right|_{\text{the } kk^{pp} \text{ locus}} = -\frac{\alpha(1-\alpha)z_t}{E_t^{pp}k_t^2}\left[E_t^{pp} - \frac{2-3\alpha}{\alpha}\right]$. Because $E_t^{pp} \in [0,1]$, as long as capital's share in production is less than labor's share, $\left.\frac{d^2 z_t}{dk_t^2}\right|_{\text{the } zz^{pp} \text{ locus}} > 0$, and the conditions for the $kk^{pp}$ locus to be decreasing and convex always hold. Because the $kk^{pp}$ locus is globally convex, as $k_t$ increases, the third intersection of the $kk^{pp}$ locus and the $zz^{pp}$ locus is not possible. So the steady state $(k^h, z^l)$, which is determined by the second intersection of the two loci, must be the desirable equilibrium.

**Appendix C. Proofs under Environmental Regulation with Green Taxes**

Proof #1. The $kk^{gt}$ locus slopes down while the $zz^{gt}$ locus slopes up in $(k_t, z_t)$ space.

Start from equations (25) and (27):

$$\Phi(y^{gt}(k_t, q), z_t) w^{gt}(k_t, q) - k_t = 0\,, \tag{C.1}$$

$$-\theta z_t + (1-\gamma q) p^{gt}(k_t, q) = 0\,. \tag{C.2}$$

Now we check the slope of the $kk^{gt}$ locus. Totally differentiating (C.1) and rearranging gives

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } kk^{gt} \text{ locus}} = -\frac{\frac{\partial \Phi_{t+1}}{\partial y_t^{gt}} \frac{\partial y_t^{gt}}{\partial k_t} w_t^{gt} + \Phi_{t+1} \frac{\partial w_t^{gt}}{\partial k_t} - 1}{\frac{\partial \Phi_{t+1}}{\partial z_t} w_t^{gt}}\,. \tag{C.3}$$

Substituting (C.1) into (C.3) and rearranging gives

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } kk^{gt} \text{ locus}} = \frac{\alpha E_t^{gt} - \beta}{(\alpha+\beta) E_t^{gt}} \frac{z_t}{k_t}\,, \tag{C.4}$$

where $E_{\Phi_{t+1}, y_t^{gt}} = \frac{\partial \Phi_{t+1}}{\partial y_t^{gt}} \frac{y_t^{gt}}{\Phi_{t+1}}$ is the elasticity of the propensity to save $\Phi_{t+1}$ with respect to income per capita $y_t^{gt}$, and $E_{\Phi_{t+1}, z_t} = \frac{\partial \Phi_{t+1}}{\partial z_t} \frac{z_t}{\Phi_{t+1}}$ is the elasticity of the propensity to save $\Phi_{t+1}$ with respect to the stock of pollution $z_t$. The elasticities are contained in the ranges $E_{\Phi_{t+1}, y_t^{gt}} \in [0,1]$ and

$E_{\Phi_{t+1},z_t} \in [-1,0]$. Again, it can be verified that $E_t^{gt} = E_{\Phi_{t+1},y_t^{gt}} = -E_{\Phi_{t+1},z_t}$. Lastly, $E_{w_t^{gt},k_t} = \frac{\partial w_t^{gt}}{\partial k_t}\frac{k_t}{w_t^{gt}}$ is the elasticity of the wage rate with respect to capital, $E_{y_t^{gt},k_t} = \frac{\partial y_t^{gt}}{\partial k_t}\frac{k_t}{y_t^{gt}}$ is the elasticity of income per capita with respect to capital, and both are equal to capital's share in production $\alpha/(\alpha+\beta)$ by (1), (3), and (4). The sufficient condition for the $kk^{gt}$ locus to slope down in $(k_t, z_t)$ space by (C.4) is Assumption (i) $0 < \alpha < \beta < 1$.

Next, we check the slope of the $zz^{gt}$ locus. Totally differentiating (C.2), using (4), and rearranging gives

$$\left.\frac{dz_t}{dk_t}\right|_{\text{the } zz^{gt} \text{ locus}} = \frac{1-\gamma q}{\theta}\frac{\alpha}{\alpha+\beta}\frac{p_t^{gt}}{k_t} > 0\,. \tag{C.5}$$

Thus, the $zz^{gt}$ locus slopes up in $(k_t, z_t)$ space.

<u>Proof #2</u>. Proposition 3.

Rewrite equations (24) and (26) as

$$k_{t+1} = \Phi\left(y^{gt}(k_t), z_t\right) w^{gt}(k_t)\,, \tag{C.6}$$

$$z_{t+1} = (1-\theta)z_t + (1-\gamma q)\,p^{gt}(k_t)\,. \tag{C.7}$$

Totally differentiating (C.6) and (C.7) around the steady state gives

$$dk_{t+1} = \left[\frac{\partial \Phi}{\partial y^{gt}}\frac{\partial y^{gt}}{\partial k} w^{gt} + \Phi \frac{\partial w^{gt}}{\partial k}\right] dk_t + \frac{\partial \Phi}{\partial z} w^{gt} dz_t,$$
$$dz_{t+1} = (1-\gamma q)\frac{\partial p^{gt}}{\partial k} dk_t + (1-\theta) dz_t.$$

Again, the $t$ subscripts are omitted to indicate that the derivatives are evaluated at the steady state. The associated Jacobian matrix is

$$J^{gt} = \begin{pmatrix} \frac{\partial \Phi}{\partial y^{gt}} \frac{\partial y^{gt}}{\partial k} w^{gt} + \Phi \frac{\partial w^{gt}}{\partial k} & \frac{\partial \Phi}{\partial z} w^{gt} \\ (1-\gamma q) \frac{\partial p^{gt}}{\partial k} & 1-\theta \end{pmatrix}. \qquad \text{(C.8)}$$

Substituting (1), (3), (4), (C.1) and (C.2) evaluated at the steady state into (C.8) and rearranging gives

$$J^{gt} = \begin{pmatrix} \frac{\alpha}{\alpha+\beta}(E^{gt}+1) & -E^{gt}\frac{k}{z} \\ \theta \frac{\alpha}{\alpha+\beta}\frac{z}{k} & 1-\theta \end{pmatrix}, \qquad \text{(C.9)}$$

where $E_{w^{gt},k} = \frac{\partial w^{gt}}{\partial k}\frac{k}{w^{gt}} = \frac{\alpha}{\alpha+\beta}$ is the elasticity of the wage rate with respect to capital,

$E_{y^{gt},k} = \frac{\partial y^{gt}}{\partial k}\frac{k}{y^{gt}} = \frac{\alpha}{\alpha+\beta}$ is the elasticity of income per capita with respect to capital,

$E_{\Phi,y^{gt}} = \frac{\partial \Phi}{\partial y^{gt}}\frac{y^{gt}}{\Phi} = E^{gt}$ is the elasticity of the propensity to save with respect to income per capita,

and $E_{\Phi,z} = \frac{\partial \Phi}{\partial z}\frac{z}{\Phi} = -E^{gt}$ is the elasticity of the propensity to save with respect to the stock of pollution. All the elasticities are evaluated at the steady state. The trace and determinant of the Jacobian matrix are

$$TrJ^{gt} = \frac{\alpha}{\alpha+\beta}\left(E^{gt}+1\right) + (1-\theta) < 2, \qquad \text{(C.10)}$$

$$DeJ^{gt} = \frac{\alpha}{\alpha+\beta}\left[(1-\theta)+E^{gt}\right] < 1. \qquad \text{(C.11)}$$

The sufficient condition for both inequalities (C.10) and (C.11) to hold is Assumption (i) $0<\alpha<\beta<1$, which implies that $0<\alpha/(\alpha+\beta)<1/2$. Under this assumption, the inequalities

(C.10) and (C.11) must hold because $E^{gt} \in [0,1]$ and $\theta \in (0,1]$. Further, we need to check the sign of the term $\left(TrJ^{gt}\right)^2 - 4DeJ^{gt}$, which establishes whether the eigenvalues have imaginary parts. From (C.10) and (C.11), we have

$$\begin{aligned}\left(TrJ^{gt}\right)^2 - 4DeJ^{gt} &= \left[\frac{\alpha}{\alpha+\beta}\left(E^{gt}+1\right)+(1-\theta)\right]^2 - 4\frac{\alpha}{\alpha+\beta}\left[(1-\theta)+E^{gt}\right] \\ &= \left[\frac{\alpha}{\alpha+\beta}\left(E^{gt}+1\right)-(1-\theta)\right]^2 - 4\theta\frac{\alpha}{\alpha+\beta}E^{gt}.\end{aligned}$$

There are two possible cases. First, if $E^{gt}$, evaluated at the unique steady state, satisfies $\left[\frac{\alpha}{\alpha+\beta}(E^{gt}+1)-(1-\theta)\right]^2 - 4\theta\frac{\alpha}{\alpha+\beta}E^{gt} < 0$, so $\left(TrJ^{gt}\right)^2 - 4DeJ^{gt} < 0$, implying that the eigenvalues are complex conjugates. With $DeJ^{gt} < 1$, we conclude that the system under green taxes will converge to the unique steady state and the convergence is cyclical. Second, if $E^{gt}$, evaluated at the unique steady state, satisfies $\left[\frac{\alpha}{\alpha+\beta}(E^{gt}+1)-(1-\theta)\right]^2 - 4\theta\frac{\alpha}{\alpha+\beta}E^{gt} > 0$, so $\left(TrJ^{gt}\right)^2 - 4DeJ^{gt} > 0$, implying that the eigenvalues are real and distinct. The characteristic polynomial is $p(\nu) = \nu^2 - \left(TrJ^{gt}\right)\nu + DeJ^{gt}$, and

$$\begin{aligned}p(1) &= 1 - TrJ^{gt} + DeJ^{gt} = \frac{\theta\beta}{\alpha+\beta} > 0, \\ p(-1) &= 1 + TrJ^{gt} + DeJ^{gt} = \left(2-\theta\right)\frac{2\alpha+\beta}{\alpha+\beta} + \frac{2\alpha}{\alpha+\beta}E^{gt} > 0.\end{aligned}$$

With $TrJ^{gt} < 2$ and $DeJ^{gt} < 1$, the system under green taxes will converge to the unique steady state and the convergence is non-cyclical.

## Appendix D. An Alternative Model with Private Healthcare

In the basic model, the representative agent treats her longevity as given. Following Bhattacharya and Qiao (2007) and Raffin and Seegmuller (2017), we consider an alternative model in which the representative agent actively engages in private healthcare efforts to improve her health status and prolong her longevity. Health status is now specified as

$$x_t = x(e_t) = \frac{e_t^{\mu} y_t^{1-\mu}}{\eta z_t}, \tag{D.1}$$

where $e_t$ is private healthcare expenditure, $y_t$ is income per capita, $z_t$ is the stock of pollution, $\eta$ measures the detrimental effect of pollution on the health status, and $\mu \in (0,1)$ is a positive parameter. Note that when the representative agent chooses private healthcare expenditure, she takes income per capita and the stock of pollution as given. The representative agent's longevity function is still given by equation (9).

To maintain tractability of the model, we assume the representative agent born in period $t$ derives utility from elderly consumption $d_{t+1}$ only. This is a similar modeling approach as Bhattacharya and Qiao (2007) and Raffin and Seegmuller (2017). The longer the representative agent lives in the elderly period, the more utility she derives from elderly consumption. This specification allows us to focus on the agent's tradeoff between private healthcare expenditure and savings, such that we can still obtain phase diagrams similar to Figure 1 and Figure 2. All else equal, increasing (decreasing) savings raises (lowers) elderly consumption, but decreases (increases) private healthcare expenditure such that the timespan when the agent can enjoy the elderly consumption is shortened (prolonged). The representative agent's lifetime utility is

$$U_t = \phi\left(x(e_t)\right)u\left(d_{t+1}\right), \quad \text{(D.2)}$$

where $u(d_{t+1})$ is assumed to be in the CRRA form, i.e., $d_{t+1}u'(d_{t+1})/u(d_{t+1}) = 1-\sigma$, and $\sigma$ is the coefficient of relative risk aversion.

The representative agent faces two budget constraints:

$$w_t = e_t + s_t, \quad \text{(D.3)}$$

$$r_{t+1}s_t = d_{t+1}. \quad \text{(D.4)}$$

The representative agent chooses private healthcare expenditure $e_t$, savings $s_t$, and elderly consumption $d_{t+1}$ to maximize her lifetime utility (D.2) subject to her young budget constraint (D.3), elderly budget constraint (D.4), health status (D.1), and longevity function (9).

Substituting $r_{t+1}s_t = d_{t+1}$ into the objective function simplifies the problem as

$$\max_{e_t, s_t} U_t = \phi\left(x(e_t)\right)u\left(r_{t+1}s_t\right)$$

$$\text{s.t.} \quad w_t = e_t + s_t.$$

The associated Lagrange is

$$L = \phi\left(x(e_t)\right)u\left(r_{t+1}s_t\right) + \psi_t\left[w_t - e_t - s_t\right],$$

where $\psi_t$ is the Lagrange multiplier of the young-age budget constraint.

The first-order conditions are

$$\phi\left(x(e_t)\right)u'\left(d_{t+1}\right)r_{t+1}-\psi_t=0, \tag{D.5}$$

$$\phi'\left(x(e_t)\right)x'(e_t)u\left(d_{t+1}\right)-\psi_t=0, \tag{D.6}$$

$$w_t-e_t-s_t=0. \tag{D.7}$$

Substituting (D.5) into (D.6) to eliminate $\psi_t$ and multiplying both sides by $s_t$ yield

$$\phi\left(x(e_t)\right)u'\left(d_{t+1}\right)d_{t+1}=s_t\phi'\left(x(e_t)\right)x'(e_t)u\left(d_{t+1}\right), \tag{D.8}$$

which states that the representative agent adjusts her young budget such that the marginal benefit of savings through its effect on elderly consumption is balanced by the marginal benefit of private healthcare expenditure through its effect on longevity. Rearranging (D.8) gives

$$\frac{s_t}{e_t}\left[\frac{x(e_t)\phi'\left(x(e_t)\right)}{\phi\left(x(e_t)\right)}\right]\left[\frac{e_t x'(e_t)}{x(e_t)}\right]=\frac{d_{t+1}u'\left(d_{t+1}\right)}{u\left(d_{t+1}\right)}. \tag{D.9}$$

Because $x(e_t)\phi'\left(x(e_t)\right)/\phi\left(x(e_t)\right)=1/\left(1+x(e_t)\right)$ by Assumption (ii) $\underline{\lambda}=0$ for simplicity, $e_t x'(e_t)/x(e_t)=\mu$, and $d_{t+1}u'(d_{t+1})/u(d_{t+1})=1-\sigma$, equation (D.9) becomes

$$s_t=\frac{1-\sigma}{\mu}e_t\left[1+x(e_t)\right]. \tag{D.10}$$

Substituting (D.3) into (D.10) to eliminate $e_t$ gives the function that determines the representative agent's savings:

$$s_t=\frac{1-\sigma}{\mu}\left(w_t-s_t\right)\left[1+x\left(w_t-s_t\right)\right]. \tag{D.11}$$

Note that in (D.11) there is no closed-form solution for $s_t$, but it is straightforward to verify that the agent's savings increase in her wage rate.

**D.1 Robustness under Environmental Regulation with Pollution Permits**

Suppose the government implements environmental regulation with pollution permits. In equilibrium, the representative agent's savings $s_t$ become next period's physical capital $k_{t+1}$. Substituting (1), (3), and (D.1) into (D.11) gives the nonlinear difference equation for capital

$$k_{t+1} = \frac{1-\sigma}{\mu}\left[\beta B\overline{p}^{1-\alpha-\beta}k_t^{\alpha} - k_{t+1}\right]\left[1 + \frac{\left(\beta B\overline{p}^{1-\alpha-\beta}k_t^{\alpha} - k_{t+1}\right)^{\mu}\left(B\overline{p}^{1-\alpha-\beta}k_t^{\alpha}\right)^{1-\mu}}{\eta z_t}\right]. \quad \text{(D.12)}$$

From equation (D.12), we define the $kk_{alternative}^{pp}$ locus where physical capital is in steady state:

$$\frac{1-\sigma}{\mu}\left[\beta B\overline{p}^{1-\alpha-\beta}k_t^{\alpha} - k_t\right]\left[1 + \frac{\left(\beta B\overline{p}^{1-\alpha-\beta}k_t^{\alpha} - k_t\right)^{\mu}\left(B\overline{p}^{1-\alpha-\beta}k_t^{\alpha}\right)^{1-\mu}}{\eta z_t}\right] - k_t = 0. \quad \text{(D.13)}$$

On the environmental side, substituting (5) into (8) and setting $\overline{p}_t = \overline{p}$ gives

$$z_{t+1} = (1-\theta)z_t - \gamma(1-\alpha-\beta)B\overline{p}^{1-\alpha-\beta}k_t^{\alpha} + \overline{p}. \quad \text{(D.14)}$$

From equation (D.14), we define the $zz_{alternative}^{pp}$ locus where the stock of pollution is in steady state:

$$-\theta z_t - \gamma(1-\alpha-\beta)B\overline{p}^{1-\alpha-\beta}k_t^{\alpha} + \overline{p} = 0. \quad \text{(D.15)}$$

The capital-environment dynamics under environmental regulation with pollution permits are determined jointly by the difference equations (D.12) and (D.14).

Multiple equilibria also emerge in this alternative model with private healthcare expenditure. The equilibrium featuring low capital and a high stock of pollution is an EPT since all combinations of capital and stock of pollution in its vicinity will gravitate toward it. The other equilibrium exhibits saddle stability. The benchmark parameters are given in Table 1 from the main paper, plus the elasticity of health status with respect to private healthcare expenditure $\mu =$ 0.5, the coefficient of relative risk aversion in elderly consumption function $\sigma = 0.9$, and the number of pollution permits $\overline{p} = 6.25$. Using these parameters, we find that the eigenvalues associated with the EPT are 0.67 and -0.07, which lie within the [-1,1] range and imply that the EPT is stable. The eigenvalues associated with the desirable equilibrium are 1.58 and -0.91. Since one is greater than 1, while the other lies within the [-1,1] range, the desirable equilibrium exhibits saddle stability. Further, the transition paths in the vicinity of the equilibria in Figure D1 confirm the dynamic properties around the equilibria. In this alternative model with private healthcare expenditure, our main results do not qualitatively change, which demonstrates the robustness of our results that environmental regulation with pollution permits might give rise to an EPT.

### D.2 Robustness under Environmental Regulation with Green Taxes

Now suppose the government implements environmental regulation with green taxes. Again, in equilibrium, the representative agent's savings become next period's capital, i.e., $s_t = k_{t+1}$. Substituting (1), (3), (4), and (D.1) into (D.11) yields

$$k_{t+1}=\frac{1-\sigma}{\mu}\left[\beta\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}-k_{t+1}\right]\times$$
$$\left[1+\frac{\left(\beta\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}-k_{t+1}\right)^{\mu}\left(\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}\right)^{1-\mu}}{\eta z_t}\right]. \quad \text{(D.16)}$$

From equation (D.16), we define the $kk^{gt}_{alternative}$ locus where physical capital is in steady state:

$$\frac{1-\sigma}{\mu}\left[\beta\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}-k_t\right]\times$$
$$\left[1+\frac{\left(\beta\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}-k_t\right)^{\mu}\left(\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1-\alpha-\beta}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}\right)^{1-\mu}}{\eta z_t}\right]-k_t=0. \quad \text{(D.17)}$$

Substituting (6) into (8) gives

$$z_{t+1}=(1-\theta)z_t+(1-\gamma q)\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}. \quad \text{(D.18)}$$

From equation (D.18), we define the $zz^{gt}_{alternative}$ locus where the stock of pollution is in steady state:

$$-\theta z_t+(1-\gamma q)\left(\tfrac{1-\alpha-\beta}{q}\right)^{\frac{1}{\alpha+\beta}}B^{\frac{1}{\alpha+\beta}}k_t^{\frac{\alpha}{\alpha+\beta}}=0. \quad \text{(D.19)}$$

The capital-environment dynamics under environmental regulation with green taxes are determined jointly by difference equations (D.16) and (D.18). All the parameters used here are the same as those used under pollution permits, except for the green-tax rate $q=0.045$. The eigenvalues associated with the equilibrium are $0.33\pm0.25i$, indicating that the equilibrium is spirally stable. The simulations shown in Figure D2 confirm that the equilibrium under a green-tax policy is spirally stable. Again, the dynamics under the green-tax system do not qualitatively change in the alternative model with private healthcare expenditure.

**Appendix E. Comparative Statics around the Steady States**

In Appendix E, we provide analytical results of the long-run impacts from the changes in both policy and technical parameters under pollution permits and under green taxes in our basic model.

Proof #1. Comparative Statics under Environmental Regulation with Pollution Permits

Equations (B.1) and (B.2) evaluated at the steady states are

$$\Phi(y^{pp}(k,\overline{p}),z,\eta)w^{pp}(k,\overline{p})-k=0, \tag{E.1}$$

$$-\theta z-\gamma a^{pp}(k,\overline{p})+\overline{p}=0. \tag{E.2}$$

Applying the Implicit Function Theorem to (E.1) and (E.2), and simplifying yields

$$\underbrace{\begin{bmatrix} \alpha\left(E^{pp}+1\right)-1 & -E^{pp}\frac{k}{z} \\ -\alpha\frac{\gamma a^{pp}}{k} & -\theta \end{bmatrix}}_{D^{pp}}\begin{bmatrix} \frac{\partial k}{\partial \overline{p}} & \frac{\partial k}{\partial \gamma} & \frac{\partial k}{\partial \eta} & \frac{\partial k}{\partial \theta} \\ \frac{\partial z}{\partial \overline{p}} & \frac{\partial z}{\partial \gamma} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \theta} \end{bmatrix}=\begin{bmatrix} -(1-\alpha-\beta)\frac{k}{\overline{p}}\left(E^{pp}+1\right) & 0 & E^{pp}\frac{k}{\eta} & 0 \\ (1-\alpha-\beta)\frac{\gamma a^{pp}}{\overline{p}}-1 & a^{pp} & 0 & z \end{bmatrix},$$

where the determinant of matrix $D^{pp}$ is

$$\left|D^{pp}\right|=-\theta\left[\alpha\left(E^{pp}+1\right)-1\right]-\alpha\left(\frac{\gamma a^{pp}}{z}\right)E^{pp}=\alpha\theta\left(\frac{1-\alpha}{\alpha}-\frac{\overline{p}}{\theta z}E^{pp}\right).$$

Notice that the sign of $\left|D^{pp}\right|$ depends on the slopes of the $kk^{pp}$ locus and the $zz^{pp}$ locus at the steady state by (B.12) and (B.14).

By Cramer's Rule, we get the effects of a change in pollution permits on steady-state capital and on the steady-state stock of pollution:

$$\frac{\partial k}{\partial \bar{p}} = \frac{\theta(\alpha+\beta)\frac{k}{\bar{p}}\left(\frac{1-\alpha-\beta}{\alpha+\beta} - \frac{\bar{p}}{\theta z}E^{pp}\right)}{\left|D^{pp}\right|}, \tag{E.3}$$

$$\frac{\partial z}{\partial \bar{p}} = \frac{\theta z}{\bar{p}} \frac{\overbrace{\left(\beta - \alpha E^{pp}\right)\frac{\bar{p}}{\theta z} + \left(1-\alpha-\beta\right)}^{(+)}}{\left|D^{pp}\right|}. \tag{E.4}$$

At the steady state $(k^l, z^h)$, $\frac{1-\alpha}{\alpha} - \frac{\bar{p}}{\theta z}E^{pp} > 0$ and $\left|D^{pp}\right| > 0$. Also notice that $\frac{1-\alpha-\beta}{\alpha+\beta} < \frac{1-\alpha}{\alpha}$. An increase in $\bar{p}$ may increase or decrease $k^l$ depending on the relative magnitudes of $\frac{1-\alpha-\beta}{\alpha+\beta}$ and $\frac{\bar{p}}{\theta z}E^{pp}$ evaluated at $(k^l, z^h)$, but unambiguously increases $z^h$. In contrast, at steady state $(k^h, z^l)$, $\frac{1-\alpha}{\alpha} - \frac{\bar{p}}{\theta z}E^{pp} < 0$ and $\left|D^{pp}\right| < 0$. An increase in $\bar{p}$ unambiguously increases $k^h$ and decreases $z^l$.

The following are the effects of changes in parameters $\gamma$, $\eta$, and $\theta$ on the steady states:

$$\frac{\partial k}{\partial \gamma} = \frac{k}{z}\frac{a^{pp}E^{pp}}{\left|D^{pp}\right|}, \tag{E.5}$$

$$\frac{\partial z}{\partial \gamma} = \frac{\alpha a^{pp}\overbrace{\left(E^{pp} - \frac{1-\alpha}{\alpha}\right)}^{(-)}}{\left|D^{pp}\right|}, \tag{E.6}$$

$$\frac{\partial k}{\partial \eta} = -\frac{\theta}{\eta}\frac{kE^{pp}}{\left|D^{pp}\right|}, \tag{E.7}$$

$$\frac{\partial z}{\partial \eta} = \frac{\alpha}{\eta}\frac{\gamma a^{pp}E^{pp}}{\left|D^{pp}\right|}, \tag{E.8}$$

$$\frac{\partial k}{\partial \theta} = \frac{kE^{pp}}{\left|D^{pp}\right|}, \tag{E.9}$$

$$\frac{\partial z}{\partial \theta} = \frac{\alpha z \overbrace{\left(E^{pp} - \frac{1-\alpha}{\alpha}\right)}^{(-)}}{\left|D^{pp}\right|}. \tag{E.10}$$

At the steady state $(k^l, z^h)$, $\left|D^{pp}\right| > 0$. Increases in $\gamma$ and $\theta$ unambiguously increase $k^l$ and decrease $z^h$. Based on the relevant expressions, both longevity and period welfare in the steady state also increase as these parameters increase. An increase in $\eta$ unambiguously decreases $k^l$ and increases $z^h$, and longevity and period welfare decrease in $\eta$. In contrast, at the steady state $(k^h, z^l)$, $\left|D^{pp}\right| < 0$. Increases in $\gamma$ and $\theta$ unambiguously decrease $k^h$ and increase $z^l$. So both longevity and period welfare in the steady state decrease as these parameters increase. An increase in $\eta$ has the opposite effects on $k^h$, $z^l$, longevity, and period welfare.

Proof #2. Comparative Statics under Environmental Regulation with Green Taxes

Equations (C.1) and (C.2) evaluated at the steady state are

$$\Phi(y^{gt}(k,q), z, \eta) w^{gt}(k,q) - k = 0, \tag{E.11}$$

$$-\theta z + (1-\gamma q)\, p^{gt}(k,q) = 0. \tag{E.12}$$

Applying the Implicit Function Theorem to (E.11) and (E.12) yields

$$\underbrace{\begin{bmatrix} \frac{\alpha}{\alpha+\beta}\left(E^{gt}+1\right)-1 & -E^{gt}\frac{k}{z} \\ \frac{\alpha}{\alpha+\beta}\frac{\theta z}{k} & -\theta \end{bmatrix}}_{D^{gt}} \begin{bmatrix} \frac{\partial k}{\partial q} & \frac{\partial k}{\partial \gamma} & \frac{\partial k}{\partial \eta} & \frac{\partial k}{\partial \theta} \\ \frac{\partial z}{\partial q} & \frac{\partial z}{\partial \gamma} & \frac{\partial z}{\partial \eta} & \frac{\partial z}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{1-\alpha-\beta}{\alpha+\beta}\frac{k}{q}\left(E^{gt}+1\right) & 0 & E^{gt}\frac{k}{\eta} & 0 \\ \frac{\theta z}{q}\left(\frac{1}{1-\gamma q}+\frac{1-\alpha-\beta}{\alpha+\beta}\right) & qp^{gt} & 0 & z \end{bmatrix},$$

where the determinant of matrix $D^{gt}$ is

$$\left|D^{gt}\right| = -\theta\left[\frac{\alpha}{\alpha+\beta}\left(E^{gt}+1\right)-1\right] + \theta\frac{\alpha}{\alpha+\beta}E^{gt} = \frac{\theta\beta}{\alpha+\beta} > 0.$$

By Cramer's Rule, we get the effects of a change in the green-tax rate on the steady-state capital and on the steady-state stock of pollution:

$$\frac{\partial k}{\partial q} = \frac{\theta k}{q(1-\gamma q)} \frac{\left[ E^{gt} - \frac{1-\alpha-\beta}{\alpha+\beta}(1-\gamma q) \right]}{\left| D^{gt} \right|}, \tag{E.13}$$

$$\frac{\partial z}{\partial q} = \frac{\alpha}{\alpha+\beta} \frac{\theta z}{(1-\gamma q)q} \frac{\overbrace{\left[ E^{gt} - \frac{\beta}{\alpha} - \frac{1-\alpha-\beta}{\alpha}(1-\gamma q) \right]}^{(-)}}{\left| D^{gt} \right|} < 0. \tag{E.14}$$

An increase in $q$ has an indeterminate effect on the steady-state capital depending on the sign of $E^{gt} - \frac{1-\alpha-\beta}{\alpha+\beta}(1-\gamma q)$, but unambiguously decreases the steady-state stock of pollution.

The effects of changes in parameters $\gamma$, $\eta$, and $\theta$ on the steady state are as follows:

$$\frac{\partial k}{\partial \gamma} = \frac{k}{z} \frac{q p^{gt} E^{gt}}{\left| D^{gt} \right|} > 0, \tag{E.15}$$

$$\frac{\partial z}{\partial \gamma} = \frac{q p^{gt} \overbrace{\left[ \frac{\alpha}{\alpha+\beta}(E^{gt}+1) - 1 \right]}^{(-)}}{\left| D^{gt} \right|} < 0, \tag{E.16}$$

$$\frac{\partial k}{\partial \eta} = -\frac{\theta k}{\eta} \frac{E^{gt}}{\left| D^{gt} \right|} < 0, \tag{E.17}$$

$$\frac{\partial z}{\partial \eta} = -\frac{\alpha}{\alpha+\beta} \frac{\theta z}{\eta} \frac{E^{gt}}{\left| D^{gt} \right|} < 0, \tag{E.18}$$

$$\frac{\partial k}{\partial \theta} = \frac{k E^{gt}}{\left| D^{gt} \right|} > 0, \tag{E.19}$$

$$\frac{\partial z}{\partial \theta} = \frac{z \overbrace{\left[ \frac{\alpha}{\alpha+\beta}(E^{gt}+1) - 1 \right]}^{(-)}}{\left| D^{gt} \right|} < 0, \tag{E.20}$$

The signs of these terms are unambiguous. Increases in $\gamma$ and $\theta$ increase the steady-state capital and decrease the steady-state stock of pollution. So longevity and period welfare in the steady state increase in $\gamma$ and $\theta$. However, an increase in $\eta$ decreases both the steady-state capital and the steady-state stock of pollution.

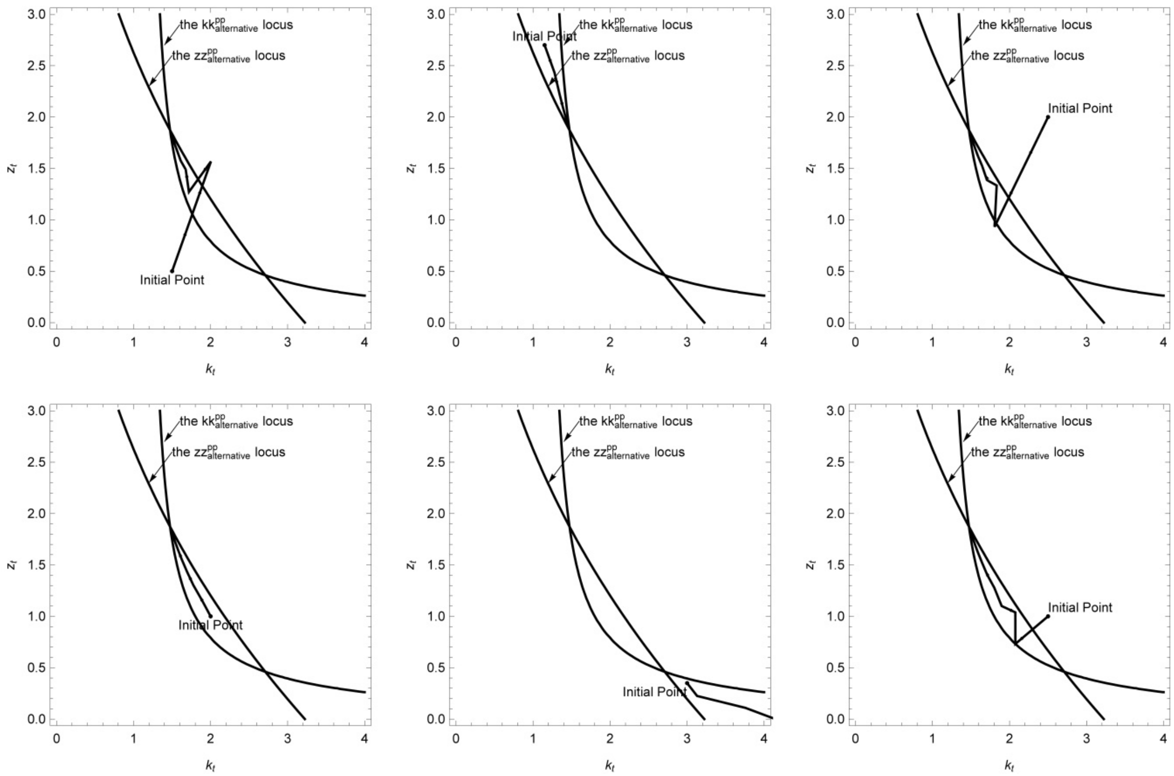


Figure D1. Dynamics under Pollution Permits with Private Healthcare Expenditure

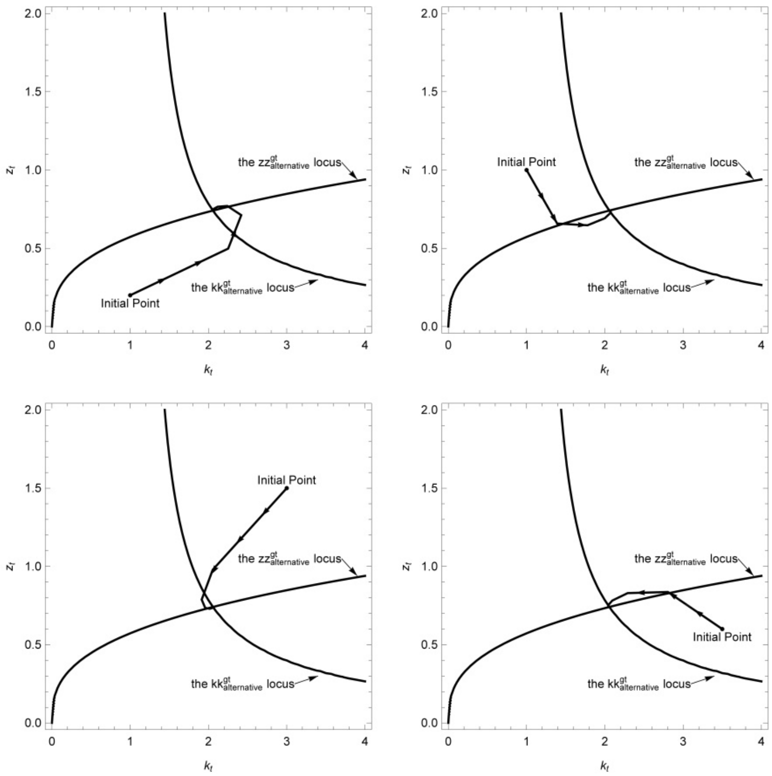


Figure D2. Dynamics under Green Taxes with Private Healthcare Expenditure

**References**

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