Physical Capital, Human Capital, and the Health Effects of Pollution in an OLG Model

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Abstract

We study the health effects of pollution on the accumulation of physical and human capital in an overlapping generations (OLG) model. Pollution negatively affects the accumulation of physical and human capital because pollution reduces longevity and the effectiveness of education expenditures. The model can generate rich dynamics that can reconcile with negative relationship between pollution and economic growth in cross-sectional data and cycles in time-series data. One interesting case is that two stable Balanced Growth Paths (BGPs) emerge with a boundary demarcating the two. One BGP is desirable featuring high economic growth and low pollution, whereas the other should be avoided because it is associated with low economic growth and high pollution. Government policy can steer the economy towards the desirable BGP. Another interesting case is that cycles may emerge causing increased economic and environmental volatility. Government policy such as changing the tax rate can eliminate the cycles.

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1 Introduction

Emerging economies, such as China and India, are often plagued by heavy pollution. Of the 100 most polluted cities in the world based on mean annual exposure to PM2.5 from 2008 to 2017, 58 are located in China and 17 are in India (World Health Organization, 2018). One may be led to the impression that economic growth and pollution generally have a positive relationship and a country can trade its environmental quality for economic growth. However, a closer look at panel data from the World Bank database (World Bank Group, 2018a,b) questions the notion that growth and pollution are positively correlated worldwide.

We collect panel data on air stock pollutants (mean annual exposure to PM2.5 and PM10 at the national level), population, and real GDP per capita.[1] Pollution is represented by the logged values of population-weighted PM2.5 and PM10, and economic growth is measured by the annual growth rate of real GDP per capita. We plot pollution against economic growth in two ways. One way of exhibiting the relationship between pollution and economic growth is with scatter plots of the countries for each year (see Figure 1 for 6 years of data on the two stock pollutants).[2] The negatively sloped trend lines indicate that some countries may experience both robust economic growth and more favorable environmental quality, while other countries may suffer from both lower economic growth and less favorable environmental quality. This is the first observation we would like to highlight. Another way of displaying the relationship is to plot the time paths of pollution and economic growth combinations for each country (see Figure 2 for selected countries and regions). Cycles frequently emerge and are depicted by the red continuous lines. This is the second observation we would like to highlight.

[Insert Figure 1 and 2 here]

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[1] The panel data cover a wide range of countries, including both developing and developed ones. The data on PM2.5 cover 190 countries and regions in 11 years (1990, 1995, 2000, 2005, 2010-2016). The data on PM10 cover 177 countries and regions in 22 years (1990-2011). The data on population and real GDP per capita go with those on PM2.5 and PM10 air pollutants.

[2] Note that due to the space limit, we do not show all of the years with data available. For PM2.5, negatively sloped trend lines appear in 9 years, accounting for 81.8% of the total years with data available. For PM10, negatively sloped trend lines appear in 11 years, accounting for 50% of the total years with data available.

What are the driving forces behind the above two observations? This paper develops an overlapping generations (OLG) model to provide a plausible explanation. The underlying mechanism we offer hinges on the interactive relationship between pollution and the ratio of physical to human capital. Through its effects on human health, pollution impacts the ratio of physical to human capital. We consider two health effects of pollution. The first health effect is that pollution reduces adults' longevity, which in turn causes adults to save less and the accumulation of physical capital is reduced. The second health effect is that pollution negatively affects children's learning, which lessens the accumulation of human capital. Since the ratio of physical to human capital also determines pollution in production and pollution abatement activities, economic growth that leads to environmental degradation causes a feedback loop.

The accumulation of physical and human capital are both hurt by a deteriorated environment, but how the ratio of physical to human capital changes in pollution and the dynamic consequences remain the key research questions. As the ratio can increase or decrease in pollution, two primary results are derived from our model. In the first, we show that two extreme Balanced Growth Paths (BGPs) and a middle one separating the previous two may emerge. The two extreme BGPs are "sinks". One is desirable in the sense that it features a high economic growth rate and a low stock of pollution, whereas the other should be avoided as it is associated with a low economic growth rate and a high stock of pollution. The BGP in the middle exhibits saddle stability, and it gives rise to a separatrix demarcating the two "sink" regions. The intuition behind the result can be explained as follows. If the negative impact of pollution on human capital accumulation is greater than that on physical capital accumulation, and worsening in the level of pollution, then we get two possible scenarios. If pollution is low, human capital becomes abundant relative to physical capital. Since physical capital is associated with pollution when employed in production, whereas human capital is clean and provides solutions to alleviating pollution, the economic growth rate is high, pollution remains at a low level, and a virtuous circle continues. In contrast, if pollution is high, human capital becomes scarce relative to physical capital. In this economy, the economic growth rate is low, pollution remains at a high level, and a vicious circle is at work. As some countries can converge to the desirable "sink" with high economic growth and low pollution, while other countries to the undesirable "sink" with low economic growth and high pollution, our first result is consistent with the empirical observation that pollution and economic growth can be negatively correlated. The policy implication is that the government should use the best available policy tool in order to steer the economy away from the undesirable BGP towards the desirable one.

The second result from our model is cycles in pollution and economic growth, where the intuition is as follows. Pollution may more adversely affect the accumulation of physical capital than that of human capital, and worsening in the stock of pollution. Suppose initially as the stock of pollution increases, the ratio of physical to human capital decreases. Because the "clean" capital becomes abundant relative to the "dirty" capital, less pollution is discharged into the environment, which in turn leads to a higher ratio of physical to human capital. Then the stock of pollution rises again. The back-and-forth dynamic relationship between the ratio of physical to human capital and pollution thus gives rise to the economic and environmental cycles. The second result replicates the empirical observation that some countries exhibit cycles in terms of pollution and economic growth. We show that government policies can be used to eliminate cycles.

We focus on the ratio of physical to human capital because the ratio is a key indicator for economic growth, with a lower ratio generally leading to higher growth (see, for example, Mulligan and Sala-i Martin, 1993; Ladrón-de Guevara et al., 1997; Barro, 2001; Duczynski, 2002, 2003). The health effects of pollution employed in our model are based on solid empirical grounds. Pollution negatively affects the accumulation of physical capital because pollution reduces longevity (see Wen and Gu, 2012; Ebenstein et al., 2015, for empirical evidence), which in turn lowers savings (see Bloom et al., 2003; Zhang and Zhang, 2005; De Nardi et al., 2009, for empirical evidence). Motivated by the empirical evidence, several theoretical papers have built models where pollution endogenously modifies agents' incentive to save through the negative effect of pollution on longevity (Pautrel, 2009; Jouvet et al., 2010; Varvarigos, 2010, 2013a; Raffin and Seegmuller, 2014; Fodha and Seegmuller, 2014). There is also empirical evidence that pollution negatively affects the accumulation of human capital because pollution increases school absences (Currie et al., 2009; Chen et al., 2018a), reduces years of schooling (Nilsson, 2009), enters and damages human brains (Maher et al., 2016), causes a significant decline in cognitive performance (Ebenstein et al., 2016), and jeopardizes mental health (Zhang et al., 2017; Kim et al., 2017; Chen et al., 2018b). The impact of pollution on human capital has also inspired other theoretical studies (see, for example, Raffin, 2012; Aloi and Tournemaine, 2013; Sapci and Shogren, 2017).

The literature cited above often deals exclusively with the effect of pollution on physical capital or with the effect of pollution on human capital. If we only focus on the negative effect of pollution on physical capital, the physical to human capital ratio unambiguously decreases in pollution. In contrast, if we only focus on the negative effect of pollution on human capital, the physical to human capital ratio unambiguously increases in pollution. It is reasonable, however, to postulate that if pollution imposes negative effects on physical and human capital in an unbalanced way, the ratio of physical to human capital may increase, decrease, or stay the same in pollution, and the subsequent dynamics and policy implications may differ from past studies. This paper thus allows pollution to have impacts on both types of capital and shows how this modeling modification can lead to rich dynamics in terms of economic and environmental consequences that match the empirical observations.

Motoyama (2016) also focuses on the dynamic interactions of pollution and the ratio of physical to human capital, which therefore merits an extensive and careful comparison with our paper. Motoyama (2016) shows that multiple equilibria may emerge in an OLG model with physical capital being the source of pollution. If the ratio of physical to human capital is less than a threshold value, the productivity of education is moderately damaged by pollution. Households invest in education and both physical and human capital accumulate. The economy converges to a low ratio of physical to human capital. However, if the ratio of physical to human capital surpasses the threshold, the productivity of education is severely damaged. Households stop investing in education and only physical capital accumulates through savings. The economy converges to a high ratio of physical to human capital. The key difference between Motoyama (2016) and this paper is twofold. First, in Motoyama (2016) pollution only negatively affects human capital. As pollution increases, the ratio of physical to human capital unambiguously rises. In contrast, we emphasize the interaction of health effects of pollution on physical capital and on human capital. We not only derive the case similar to Motoyama (2016), but also show that if the ratio of physical to human capital decreases in pollution, a possibility absent from Motoyama (2016), economic and environmental cycles may emerge. We also show that the government has a role in eliminating the volatility caused by the cycles. Second, in Motoyama (2016) the government does not invest in education and pollution is implicitly modeled as being associated with physical capital. In our model, however, the government provides public education even if agents may not invest in private education. We also explicitly model the dynamics for the stock of pollution where unlimited growth of pollution can be checked by pollution abatement financed by the government spending. Our model thus allows us to discuss policy implications.

The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 lays out the equilibria and derives difference equations representing the economic and environmental dynamic systems. Section 4 analyzes the balanced growth path and the local dynamic properties. Section 5 provides numerical examples to show that there are two interesting cases where government policies are required either to eliminate economic and environmental cycles or to steer the economy away from the less favorable balanced growth path. Section 6 concludes.

2 The Model

2.1 Firms

The production factors in this model are physical capital K_t, and labor L_t augmented by human capital H_t. Denote r_t as the rental price of physical capital, and w_t as wage rate paid per unit of labor. The production function that a typical competitive firm employs to produce a final good is $Y_t = AK_t^{\alpha}(H_tL_t)^{1-\alpha}$, where $A > 0$ is a production function scalar, $\alpha \in (0,1)$ is physical capital's share in production, and $1-\alpha$ is augmented labor's share in production. The price of the final good is normalized to 1. The firm pays a proportional tax, τ, on the final good to the government (Barro, 1990; Devarajan et al., 1996; Agénor and Neanidis, 2011). In each period, the representative firm hires physical capital K_t and labor L_t to maximize its profits, so the profit-maximization problem remains the same in each period:

$$\max_{K_t,L_t} \pi_t = (1-\tau)AK_t^{\alpha}(H_tL_t)^{1-\alpha} - r_tK_t - w_tL_t.$$

Define $k_t = {}^{K_t}/_{H_t}$ as the ratio of physical to human capital. Since each input is paid its marginal product, the first-order conditions are

$$r_t = (1-\tau)\alpha Ak_t^{\alpha-1}L_t^{1-\alpha}, \tag{1}$$

$$w_t = (1-\tau)(1-\alpha)Ak_t^{\alpha}H_tL_t^{-\alpha}. \tag{2}$$

2.2 Government

The government collects fiscal revenues through the proportional tax on the final good, $\tau AK_t^{\alpha}(H_tL_t)^{1-\alpha} = \tau Ak_t^{\alpha}H_tL_t^{1-\alpha}$. The government allocates a portion of the fiscal revenues, $\Delta \in [0,1]$, to finance pollution abatement a_t, and the remaining portion, $1-\Delta \in [0,1]$, to finance public education m_t.[3] The government runs a balanced budget in each period, which requires

$$a_t = \Delta\tau Ak_t^{\alpha}H_tL_t^{1-\alpha}, \tag{3}$$

$$m_t = (1-\Delta)\tau Ak_t^{\alpha}H_tL_t^{1-\alpha}. \tag{4}$$

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[3] The government may also allocate fiscal revenues to other public uses, such as infrastructure that would enhance the physical capital (see, for example, Agénor, 2011). This additional use of fiscal revenues diverts resources away from pollution abatement and public education, both of which support the accumulation of human capital. So introducing government spending on infrastructure will increase the ratio of physical to human capital relative to the extant model. Because the primary focus of this paper is on how the health effects of pollution influence the ratio of physical to human capital, we abstract from the public expenditures on infrastructure.

2.3 Stock of Pollution

The stock of pollution increases due to production activities but decreases due to public pollution abatement. Because it is "too good to be true" that emissions cease to grow (Economides and Philippopoulos, 2008), we assume that the unabated flow of pollution, ρK_t, are proportional to physical capital, where $\rho > 0$ represents the polluting capacity of physical capital. So as long as physical capital grows, unabated pollution increases. We also specify that the abated flow of pollution is the ratio of unabated pollution to public pollution abatement, $\rho K_t / a_t$ (see, for example, Gradus and Smulders, 1993; Smulders and Gradus, 1996; Pautrel, 2009, 2012).[4] An implicit assumption is $a_t > 1$, such that the abated flow of pollution cannot surpass the unabated flow of pollution. The stock of pollution, z_t, evolves according to

$$z_{t+1} = (1-\theta)z_t + \frac{\rho K_t}{a_t}, \tag{5}$$

where $\theta \in (0,1)$ represents the dissipation rate of the stock of pollution. The dynamics for the stock of pollution (5) implies that when there are no human activities, the stock of pollution converges to zero in the long run. The stock of pollution adversely affects the economy by imposing two types of health effects on the representative agent, which will be fully explained in the following section.

2.4 Agents

The representative agent lives three periods, i.e., childhood, adulthood, and elderhood. In childhood, the agent receives education to accumulate human capital. In adulthood, the agent gives birth to one child, inelastically supplies one unit of labor to earn wage income, and makes

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[4] There are two advantages associated with this specification. First, Gradus and Smulders (1993) show that even when investment activities (e.g., the use of cleaner fuels, which allows for a reduction in the amount of pollution per unit of capital in the production process) and abatement activities (e.g., "end-of-pipe measures", which aim at cleaning up existing pollution) are distinguished, this function for net emissions still qualitatively holds. So although we use the term "abatement", we cover both cases of reducing the flow of pollution and the existing stock of pollution. Second, Pautrel (2012) argues that the linear specification of the net emissions (for example, $\rho k_t - a_t$) is "not constant along the Balanced Growth Path (BGP), and therefore the stock of pollution explodes in the long run."

decisions in terms of consumption during adulthood, savings, and private education expenditures on her child to maximize lifetime utility. In elderhood, the agent enjoys the fruits due to the decisions made during adulthood, i.e., elderly consumption financed by her savings and her child's human capital because of altruism. The agent lives the entirety of her childhood and adulthood, but lives only a fraction of her elderhood, ϕ. The representative agent born at the beginning of period $t-1$ thus has a longevity equal to $2+\phi_{t+1}$. The representative agent treats her longevity as given. Similar to Varvarigos (2013b) and Fodha and Seegmuller (2014), we specify that the fraction of elderhood that an agent lives depends on the stock of pollution. Specifically, for a representative agent born at the beginning of period $t-1$, her elderly longevity depends on the stock of pollution during her adulthood, i.e., $\phi_{t+1} = \phi(z_t) \in [\underline{\phi}, \overline{\phi}]$, where $\underline{\phi} \geq 0$ and $\overline{\phi} \leq 1$ are the lower and upper bounds of longevity. We assume $\phi(0) = \overline{\phi}$, $\phi(z_t) \to \underline{\phi}$ for $z_t \to +\infty$, and $\phi'(z_t) < 0$. The specification of longevity as a function of the stock of pollution captures the first type of health effect imposed by pollution on longevity.[5]

Following Raffin (2012), we introduce the effectiveness of education expenditures $\lambda_t = \lambda(z_t) \in [\underline{\lambda}, \overline{\lambda}]$, where $\underline{\lambda} \geq 0$ and $\overline{\lambda} \leq 1$ are the lower and upper bounds of the effectiveness of education expenditures. We assume $\lambda(0) = \overline{\lambda}$, $\lambda(z_t) \to \underline{\lambda}$ for $z_t \to +\infty$, and $\lambda'(z_t) < 0$. The effectiveness of education expenditures depends on the stock of pollution because a worsened environmental quality reduces learning time due to absenteeism, undermines cognitive performance, or damages a child's mental health and nervous system (lead pollution, for example). The effectiveness of education expenditures as a function of pollution thus captures the second type of health effect of pollution on learning. Denote e_t as private education expenditures. Total education expenditures are $\mu m_t + e_t$, where $\mu > 0$ measures the relative strength of public to private education expenditures (Buiter and Kletzer, 1995; Osang and Sarkar, 2008). One can think of total education expenditures as the sum of what the government pays for compulsory

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[5] It is true that agents can actively affect their own longevity, but introducing the idea of endogenous risk may not be consistent with the Balanced Growth Path (BGP), which requires the expressions revolve around the ratio of physical to human capital. The reason of emphasizing the BGP at the cost of endogenous risk is that human capital serves as an engine of growth in the model. On the BGP, the ratio of physical to human capital remains constant, but physical capital and human capital both grow at the same rate. It is common to consider that longevity solely depends on environmental quality, see, for example, Varvarigos (2013b) and Fodha and Seegmuller (2014).

education, plus what the parents choose to pay in the form of college tuition and fees for her child. With $\lambda(z_t)$ adjusting total education expenditures, effective education expenditures thus become $\lambda(z_t)(\mu g_t + e_t)$. Besides education expenditures, the evolution of human capital also depends on parents' human capital (e.g., parental example and guidance). Both effective education expenditures and parents' human capital are subject to constant returns to scale. As the agent born at the beginning of period $t-1$ has human capital H_t in period t and gives birth to a child at the beginning of period t, the child born at the beginning of period t has human capital in period $t+1$ equal to

$$H_{t+1} = B\left[\lambda_t\left(\mu m_t + e_t\right)\right]^{\beta} H_t^{1-\beta}, \tag{6}$$

where $B > 0$ is a scalar, and $\beta \in (0,1)$ is the share of effective education expenditures in the formation of human capital.

Taking her longevity ϕ_{t+1} and human capital H_t as given, the agent born at the beginning of period $t-1$ makes decisions at the beginning of period t. The agent derives utility from her adulthood consumption c_t, elderhood consumption d_{t+1}, and her child's stock of human capital H_{t+1} because of altruism (Osang and Sarkar, 2008). Assuming a logarithmic function that is additively separable, the agent's lifetime utility is

$$U_{t-1} = \ln c_t + \phi_{t+1}\left[\ln d_{t+1} + \chi \ln H_{t+1}\right], \tag{7}$$

where the parameter $\chi > 0$ represents the agent's altruism towards her child's human capital. In equation (7), both elderly consumption and child's human capital are discounted by longevity, ϕ_{t+1}. So an agent's altruism towards her child's human capital is adjusted by longevity ϕ_{t+1} and the effective altruism becomes $\phi_{t+1}\chi$. The adjustment of altruism by longevity implies that, aside from the effectiveness of education expenditures $\lambda(z_t)$, the stock of pollution also influences the accumulation of human capital through longevity $\phi(z_t)$.

During adulthood, the representative agent uses her wage income w_t to cover her adulthood consumption c_t, savings s_t, and private education expenditures for her child e_t. When the representative agent is old, she uses the remunerated savings, $r_{t+1}s_t$, to finance her elderly consumption, d_{t+1}. So the representative agent faces two budget constraints – one for her adulthood and one for her elderhood:

$$w_t = c_t + s_t + e_t, \tag{8}$$

$$r_{t+1}s_t = d_{t+1}. \tag{9}$$

The representative agent's problem is to maximize (7) by choosing c_t, s_t, e_t, and d_{t+1} subject to (6), (8), (9), as well as an additional non-negative constraint $e_t \geq 0$.

The agent may or may not invest in private education based on the Kuhn-Tucker conditions. If the agent invests positively in private education, $e_t > 0$, the consumption function, savings function, and private education expenditures function are

$$c_t = \frac{\Phi_{t+1}}{\phi_{t+1}}\left(w_t + \mu m_t\right), \tag{10}$$

$$s_t = \Phi_{t+1}\left(w_t + \mu m_t\right), \tag{11}$$

$$e_t = \chi\beta\Phi_{t+1}(w_t + \mu m_t) - \mu m_t, \tag{12}$$

where $\Phi_{t+1} = \frac{1}{1+\chi\beta+\left(1/\phi_{t+1}\right)}$ is the agent's propensity to save when $e_t > 0$, which satisfies $\Phi'(z_t) < 0$ and $\Phi_{t+1} \in \left[\frac{\underline{\phi}}{(1+\chi\beta)\underline{\phi}+1}, \frac{\overline{\phi}}{(1+\chi\beta)\overline{\phi}+1}\right]$. Other things equal, if the agent lives for a shorter time (ϕ_{t+1} is smaller), she increases adulthood consumption, reduces savings that finance elderhood consumption, and reduces private education expenditures.

However, if the following condition holds:

$$\frac{\chi\beta\phi_{t+1}}{\mu m_t} < \frac{1}{w_t - s_t}, \tag{13}$$

the agent does not invest in private education, i.e., $e_t = 0$. Condition (13) says that if the marginal utility gained from the first dollar invested in private education is smaller than the utility lost due to the foregone young consumption, the agent does not invest in private education. The stock of pollution plays a role in modifying the agent's decision to invest in private education because pollution reduces longevity, rendering the marginal utility gained from the first dollar invested in private education even smaller. As the representative agent does not invest in private education, the accumulation of human capital only depends on public education expenditures. The consumption function and savings function become

$$\begin{aligned} c_t &= \frac{\overline{\Phi}_{t+1}}{\phi_{t+1}} w_t, &(14)\\ s_t &= \overline{\Phi}_{t+1} w_t, &(15) \end{aligned}$$

where $\overline{\Phi}_{t+1} = \frac{1}{1+\left(1/\phi_{t+1}\right)}$ is the agent's propensity to save when $e_t = 0$, which satisfies $\overline{\Phi}'(z_t) < 0$ and $\overline{\Phi}_{t+1} \in \left[\frac{\underline{\phi}}{1+\underline{\phi}}, \frac{\overline{\phi}}{1+\overline{\phi}}\right]$. All else equal, if the agent lives for a shorter time, she increases adulthood consumption and cuts back on savings.

3 Equilibria

Because the representative agent is assumed to live the entirety of her adulthood, for simplicity we normalize the labor size to unity, $L_t = 1$. We first describe the dynamics of the environment. Substituting (3) into (5) gives the difference equation for the stock of pollution:

$$z_{t+1} = (1-\theta) z_t + \frac{\rho}{\Delta \tau A} k_t^{1-\alpha}. \tag{16}$$

From Equation (16), taking the first difference of the stock of pollution and setting it equal to zero yields the zz locus:

$$-\theta z_t + \frac{\rho}{\Delta \tau A} k_t^{1-\alpha} = 0. \tag{17}$$

The zz locus defines all the combinations of k_t and z_t where the stock of pollution is in steady state. On the zz locus, the stock of pollution increases in the ratio of physical to human capital because human capital is assumed to be "clean" and physical capital "dirty", which implies that an economy with abundant physical capital relative to human capital tends to have a higher stock of pollution.

We next describe the dynamics of the economy. Whether or not the representative agent invests in private education alters the accumulations of physical and human capital. We hereafter denote *PE* (Private Education) as the regime where the agent's investment in private education is positive, $e_t > 0$, and *NPE* (No Private Education) as the regime where $e_t = 0$. We can show that the combination of policy parameters, τ and Δ, dictates the representative agent's incentive to invest in private education.

Proposition 1. *(i) The PE regime: if* $\Delta \geq 1-(1-\alpha)\frac{\underline{\phi}}{1+\underline{\phi}}\frac{\chi\beta}{\mu}\frac{1-\tau}{\tau} \equiv f_1(\tau)$, *where* $f_1(\tau)$ *satisfies* $f_1'(\tau) > 0$, $f_1''(\tau) < 0$, *and solving* $f_1(\tau) = 0$ *yields* $\tilde{\tau} = (1-\alpha)\frac{\underline{\phi}}{1+\underline{\phi}}\frac{\chi\beta}{\mu} \Big/ \left[1+(1-\alpha)\frac{\underline{\phi}}{1+\underline{\phi}}\frac{\chi\beta}{\mu}\right]$, *for any stock of pollution* $z_t \in [0, +\infty)$, *the agent's investment in private education is always positive,* $e_t > 0$.

(ii) The NPE regime: if $\Delta \leq 1-(1-\alpha)\frac{\overline{\phi}}{1+\overline{\phi}}\frac{\chi\beta}{\mu}\frac{1-\tau}{\tau} \equiv f_2(\tau)$, *where* $f_2(\tau)$ *satisfies* $f_2'(\tau) > 0$, $f_2''(\tau) < 0$, *and solving* $f_2(\tau) = 0$ *yields* $\hat{\tau} = (1-\alpha)\frac{\overline{\phi}}{1+\overline{\phi}}\frac{\chi\beta}{\mu} \Big/ \left[1+(1-\alpha)\frac{\overline{\phi}}{1+\overline{\phi}}\frac{\chi\beta}{\mu}\right]$, *for any stock of pollution* $z_t \in [0, +\infty)$, *the agent's investment in private education is always zero,* $e_t = 0$.

(iii) Both the PE and NPE regimes: if $f_2(\tau) < \Delta < f_1(\tau)$, *there exists a threshold stock of pollution* $z^o(\tau, \Delta)$ *that is implicitly defined by* $\frac{\phi(z^o)}{1+\phi(z^o)} = \frac{1}{\chi\beta}\frac{\mu\tau(1-\Delta)}{(1-\tau)(1-\alpha)}$. *When* $z_t \in [0, z^o]$, *the agent's investment in private education is positive,* $e_t > 0$*; when* $z_t \in (z^o, +\infty)$, *the agent's investment in private education is zero,* $e_t = 0$.

Proof. See Appendix A. □

Proposition 1 indicates that the policy set is divided into three regions and this fact is more clearly revealed in Figure 3. Region I corresponds to the *PE* regime, where the policy combinations of τ and Δ lead the representative agent to invest in private education irrespective of the stock of pollution. Region II corresponds to the *NPE* regime, where the agent never invests in private education. Region III corresponds to both the *PE* and *NPE* regimes, where the agent's decision on private education is determined by the threshold stock of pollution, $z^o(\tau, \Delta)$.

[Insert Figure 3 here]

Because this period's savings become next period's physical capital, we have $s_t = K_{t+1}$. Substituting expressions into the savings function (11) when $e_t > 0$ and into the savings function (15) when $e_t = 0$ yields the difference equations for physical capital under the *PE* and *NPE* regimes:

$$PE: \frac{K_{t+1}}{K_t} = A\Phi(z_t)\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]k_t^{\alpha-1}, \tag{18a}$$

$$NPE: \frac{K_{t+1}}{K_t} = A\overline{\Phi}(z_t)(1-\tau)(1-\alpha)k_t^{\alpha-1}. \tag{18b}$$

The accumulation of physical capital is hampered by the stock of pollution because pollution reduces agent's longevity and thus her propensity to save both under the *PE* regime and under the *NPE* regime, i.e., $\Phi'(z_t) < 0$ and $\overline{\Phi}'(z_t) < 0$.

Substituting (4) and (12) into (6) when $e_t > 0$ and substituting (4) into (6) when $e_t = 0$ yields the difference equations for human capital under the *PE* and *NPE* regimes:

$$PE: \frac{H_{t+1}}{H_t} = B\left[\chi\beta A\Phi(z_t)\lambda(z_t)\right]^{\beta}\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]^{\beta}k_t^{\alpha\beta}, \tag{19a}$$

$$NPE: \frac{H_{t+1}}{H_t} = B\left[A\lambda(z_t)\right]^{\beta}\left[\mu\tau(1-\Delta)\right]^{\beta}k_t^{\alpha\beta}. \tag{19b}$$

The accumulation of human capital is also reduced by the stock of pollution, but the channels are slightly different under the *PE* and *NPE* regimes. Under the *PE* regime, pollution reduces the effectiveness of education expenditures, $\lambda'(z_t) < 0$. Pollution also reduces longevity, which in turn decreases the agent's effective altruism, so the agent invests less in private education. This fact is reflected by the term $\Phi'(z_t) < 0$. Under the *NPE* regime, in contrast, the agent does not invest in private education, and thus the term $\overline{\Phi}(z_t)$ does not enter equation (19b). However, pollution also weakens the effectiveness of education expenditures, but only through the public education expenditures channel.

Now we are ready to derive the evolution of the ratio of physical to human capital.

Because ${}^{k_{t+1}}/_{k_t} = {}^{(K_{t+1}/K_t)}/_{(H_{t+1}/H_t)}$, using the dynamics for physical and human capital under the *PE* and *NPE* regimes, i.e., dividing (18a) by (19a) and dividing (18b) by (19b) yields the difference equations in terms of the ratio of physical to human capital:

$$PE: \; k_{t+1} \;=\; \frac{A^{1-\beta}}{B(\chi\beta)^{\beta}}\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]^{1-\beta}\frac{\Phi(z_t)^{1-\beta}}{\lambda(z_t)^{\beta}}k_t^{\alpha(1-\beta)}, \tag{20a}$$

$$NPE: \; k_{t+1} \;=\; \frac{A^{1-\beta}}{B}\frac{(1-\tau)(1-\alpha)}{[\mu\tau(1-\Delta)]^{\beta}}\frac{\overline{\Phi}(z_t)}{\lambda(z_t)^{\beta}}k_t^{\alpha(1-\beta)}. \tag{20b}$$

From equations (20a) and (20b), taking the first difference of the ratios of physical to human capital and setting them equal to zeros gives the *kk* loci under the *PE* and *NPE* regimes:

$$PE: \; \frac{A^{1-\beta}}{B(\chi\beta)^{\beta}}\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]^{1-\beta}\frac{\Phi(z_t)^{1-\beta}}{\lambda(z_t)^{\beta}}k_t^{\alpha(1-\beta)}-k_t \;=\; 0, \tag{21a}$$

$$NPE: \; \frac{A^{1-\beta}}{B}\frac{(1-\tau)(1-\alpha)}{[\mu\tau(1-\Delta)]^{\beta}}\frac{\overline{\Phi}(z_t)}{\lambda(z_t)^{\beta}}k_t^{\alpha(1-\beta)}-k_t \;=\; 0. \tag{21b}$$

The *kk* loci define all the combinations of k_t and z_t where the ratios of physical to human capital are in steady state. On the *kk* loci, the ratio of physical to human capital may increase or decrease in the stock of pollution. The reason lies in the asymmetry of the health effects of pollution on the accumulation of physical capital and on the accumulation of human capital. This asymmetry is revealed by the relative magnitudes of the negative marginal effects of pollution. Note that the slopes of the *kk* loci do not depend on how pollution affects the levels but the growth rates of physical and human capital. The conditions for the *kk* loci to slope up or down are summarized in the following proposition.

Proposition 2. *Define* $g_K = \ln\left({}^{K_{t+1}}/_{K_t}\right)$ *as the growth rate of physical capital, and* $g_H = \ln\left({}^{H_{t+1}}/_{H_t}\right)$ *as the growth rate of human capital. Whether it is under the PE regime or under the NPE regime, if the negative marginal effect of pollution on the growth rate of human capital is larger (smaller) than that on the growth rate of physical capital, the kk locus slopes up (down) in* (z_t, k_t) *space.*

Proof. See Appendix B. □

Proposition 2 makes intuitive sense in that if the accumulation of human capital is more negatively affected by pollution than that of physical capital, physical capital becomes abundant relative to human capital. The ratio of physical to human capital rises and the *kk* locus slopes up in (z_t, k_t) space. In contrast, if the accumulation of human capital is less negatively affected by pollution than that of physical capital, human capital becomes abundant relative to physical capital. The ratio of physical to human capital falls and the *kk* locus slopes down in (z_t, k_t) space.

To complete our description of the dynamic system for the economy and the environment, we check the continuity and slopes of the two *kk* loci under the *PE* and *NPE* regimes at the threshold stock of pollution, $z^o(\tau, \Delta)$, provided that this threshold exists based on the fact established in Proposition 1. The results are summarized in the following proposition.

Proposition 3. *Suppose the threshold stock of pollution* $z^o(\tau, \Delta)$ *exists according to Proposition 1. At the threshold stock of pollution,*

(i) the values for the ratio of physical to human capital determined by the kk loci under the PE and NPE regimes are equal, so there is no discontinuity between the two kk loci under the PE and NPE regimes;

(ii) the slope of the kk locus under the PE regime is larger than that of the kk locus under the NPE regime.

Proof. See Appendix C. □

The first part of Proposition 3 is a mathematical fact. The second part highlights the difference in the slopes of the *kk* loci when the regime switches. The reason is that the propensity to save is more sensitive to the change in the stock of pollution under the *NPE* regime than under the *PE* regime. So when the regime switches from *PE* to *NPE*, the slope of the *kk* locus becomes smaller.

4 Balanced Growth Path and Local Dynamics

We have thus far derived the zz locus, as well as the kk loci both under the PE regime and under the NPE regime. These loci are the building blocks for the Balanced Growth Path (BGP) and its local dynamics. A BGP is represented by the intersection of the kk locus and the zz locus. On the BGP, the ratio of physical to human capital k_t, the stock of pollution z_t, longevity $\phi(z_t)$, and the effectiveness of education expenditures $\lambda(z_t)$ remain constant, while physical capital K_t, human capital H_t, final output Y_t, young consumption c_t, and elderly consumption d_t all grow at the same rate. Note that the zz locus may intersect with the kk locus under the PE regime and/or under the NPE regime, so denote z_i^* as the stock of pollution and k_i^* as the ratio of physical to human capital on the BGP, where $i = PE, NPE$. The rest of the variables and the growth rate on the BGP can be derived based on z_i^* and k_i^*. Also denote g_i^* as the growth rate on the BGP. The values for g_i^* under the PE regime and under the NPE regime can be derived as follows.

Under the PE regime, substituting (17) in (21a) to eliminate k_t gives

$$\frac{A^{1-\beta}}{B(\chi\beta)^{\beta}}\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]^{1-\beta}\frac{\Phi(z_{PE}^*)^{1-\beta}}{\lambda(z_{PE}^*)^{\beta}}=\left(\frac{\Delta\tau A\theta}{\rho}z_{PE}^*\right)^{\frac{1-\alpha(1-\beta)}{1-\alpha}}, \tag{22}$$

which implicitly defines the stock of pollution on the BGP, z_{PE}^*. Substituting z_{PE}^* into (17) gives the ratio of physical to human capital on the BGP:

$$k_{PE}^*=\left(\frac{\Delta\tau A\theta}{\rho}z_{PE}^*\right)^{\frac{1}{1-\alpha}}. \tag{23}$$

Substituting k_{PE}^* and z_{PE}^* into either (18a) or (19a) and taking natural logs gives the growth rate on the BGP under the PE regime:

$$g_{PE}^*=\ln\left\{\left[(1-\tau)(1-\alpha)+\mu\tau(1-\Delta)\right]\frac{\rho}{\Delta\tau\theta}\frac{\Phi(z_{PE}^*)}{z_{PE}^*}\right\}. \tag{24}$$

Similarly, the growth rate on the BGP under the *NPE* regime is

$$g^*_{NPE} = \ln\left[(1-\tau)(1-\alpha)\frac{\rho}{\Delta\tau\theta}\frac{\overline{\Phi}(z^*_{NPE})}{z^*_{NPE}}\right]. \tag{25}$$

The local dynamics surrounding a BGP depend on the *zz* locus intersecting with the *kk* locus under the *PE* regime or under the *NPE* regime, and on the relative slopes of the two loci. If the intersection of the *zz* locus and the *kk* locus happens under the *PE* regime, the agent invests in private education, and equations (20a) and (16) dictate the local dynamics of the BGP. We summarize the local dynamic properties of the BGP in the following proposition.

Proposition 4. *Under the PE regime, the representative agent invests in private education, i.e.,* $e_t > 0$. *Let* $E_{\Phi,z}$ *be the elasticity of the propensity to save with respect to the stock of pollution, and* $E_{\lambda,z}$ *the elasticity of the effectiveness of education expenditures with respect to the stock of pollution. Both elasticities are evaluated on the BGP.*

(i) The kk locus slopes up on the BGP. If the kk locus is flatter than the zz locus on the BGP, the BGP is locally stable; if the kk locus is steeper than the zz locus on the BGP, the BGP exhibits locally saddle stability when $1-\alpha(1-\beta) < (1-\alpha)\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right] < \left(\frac{2}{\theta}-1\right)[1+\alpha(1-\beta)]$, *and the BGP is locally unstable when* $1-\alpha(1-\beta) < \left(\frac{2}{\theta}-1\right)[1+\alpha(1-\beta)] < (1-\alpha)\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right]$.

(ii) The kk locus slopes down on the BGP. If $[\alpha(1-\beta)-(1-\theta)]^2 + 4(1-\alpha)\theta\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right] > 0$ *and* $\alpha(1-\beta)(1-\theta) - (1-\alpha)\theta\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right] < 1$, *the BGP is locally stable; if* $[\alpha(1-\beta)-(1-\theta)]^2 + 4(1-\alpha)\theta\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right] < 0$, *the BGP features locally dampened cycles when* $\alpha(1-\beta)(1-\theta) - (1-\alpha)\theta\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right] < 1$, *and the BGP features locally outward cycles when* $\alpha(1-\beta)(1-\theta) - (1-\alpha)\theta\left[(1-\beta)E_{\Phi,z} - \beta E_{\lambda,z}\right] > 1$.

Proof. See Appendix D. ☐

However, if the *zz* locus intersects with the *kk* locus under the *NPE* regime, the agent does not invest in private education and equations (20b) and (16) dictate the local dynamics. We summarize the local dynamic properties of the BGP in the following proposition.

Proposition 5. *Under the NPE regime, the agent does not invest in private education, i.e.,* $e_t = 0$*. Let* $E_{\overline{\Phi},z}$ *be the elasticity of the propensity to save with respect to the stock of pollution evaluated on the BGP.*

(i) The kk locus slopes up on the BGP. If the kk locus is flatter than the zz locus on the BGP, the BGP is locally stable; if the kk locus is steeper than the zz locus on the BGP, the BGP exhibits locally saddle stability when $1-\alpha(1-\beta) < (1-\alpha)\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right) < \left(\frac{2}{\theta}-1\right)[1+\alpha(1-\beta)]$*, and the BGP is locally unstable when* $1-\alpha(1-\beta) < \left(\frac{2}{\theta}-1\right)[1+\alpha(1-\beta)] < (1-\alpha)\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right)$*.*

(ii) The kk locus slopes down on the BGP. If $[\alpha(1-\beta)-(1-\theta)]^2 + 4(1-\alpha)\theta\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right) > 0$ *and* $\alpha(1-\beta)(1-\theta) - (1-\alpha)\theta\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right) < 1$*, the BGP is locally stable; if* $[\alpha(1-\beta)-(1-\theta)]^2 + 4(1-\alpha)\theta\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right) < 0$*, the BGP features locally dampened cycles when* $\alpha(1-\beta)(1-\theta) - (1-\alpha)\theta\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right) < 1$*, and the BGP features locally outward cycles when* $\alpha(1-\beta)(1-\theta) - (1-\alpha)\theta\left(E_{\overline{\Phi},z} - \beta E_{\lambda,z}\right) > 1$*.*

Proof. See Appendix E. ☐

5 Numerical Examples

Based on the basic properties of the longevity function $\phi(z_t)$ and the education expenditures effectiveness function $\lambda(z_t)$, we have analytically derived the slopes of the *kk* loci under the *PE* and *NPE* regimes and the local dynamic properties of the BGP. However, there is a lack of empirical evidence on how exactly pollution reduces longevity and the effectiveness of education expenditures. So how the curvature of the *kk* locus is related to the shapes of $\phi(z_t)$ and $\lambda(z_t)$ is a primary interest of our research. To proceed, we adopt flexible functional forms that satisfy the basic properties and encompass different shapes of $\phi(z_t)$ and $\lambda(z_t)$. The longevity function is $\phi(z_t) = \left(\overline{\phi} + \underline{\phi} z_t^{c_\phi}\right) \Big/ \left(1 + z_t^{c_\phi}\right)$, where $\overline{\phi}$ and $\underline{\phi}$ are the upper and lower bounds of longevity, and $c_\phi > 0$ is a curvature parameter. The education expenditures effectiveness function is $\lambda(z_t) = \left(\overline{\lambda} + \underline{\lambda} z_t^{c_\lambda}\right) \Big/ \left(1 + z_t^{c_\lambda}\right)$, where $\overline{\lambda}$ and $\underline{\lambda}$ are the upper and lower bounds of the education expenditures effectiveness, and $c_\lambda > 0$ is a curvature parameter. These functional forms are general enough to allow for a wide range of cases for the negative health effects of pollution. We next use numerical examples to show how the curvature of the *kk* locus is related to the relative shapes of longevity $\phi(z_t)$ and the effectiveness of education expenditures $\lambda(z_t)$, and how our model is consistent with the empirical observations illustrated in Figure 1 and 2. To systematically accomplish this, we introduce a parameter ε to represent the differential between the two curvature parameters, i.e., $\varepsilon = c_\lambda - c_\phi$. We fix the curvature of the longevity function c_ϕ and vary the value for ε, such that we can experiment with different degrees of curvature for the effectiveness of education expenditures c_λ. The benchmark parameters used in the following numerical examples are listed in Table 1.

[Insert Table 1 here]

Example 1, $\varepsilon = 0$. The longevity function is $\phi(z_t) = \frac{1}{1+z_t^3}$ and the education expenditures effectiveness function is $\lambda(z_t) = \frac{1}{1+z_t^3}$. The upper panel of Figure 4 illustrates the shapes of $\phi(z_t)$ and $\lambda(z_t)$. The lower panel of Figure 4 shows that there exists a threshold stock of pollution and the *zz* locus intersects with the *kk* locus under the *PE* regime. The *kk* locus under the *PE* regime slopes up and is flatter than the *zz* locus at the intersection. The BGP is locally stable, and the associated eigenvalues are 0.55 and -0.22.

[Insert Figure 4 here]

In Example 1, although the shapes of $\phi(z_t)$ and $\lambda(z_t)$ are the same, the *kk* locus increases in the stock of pollution under the *PE* regime. This is because under the *PE* regime, the negative marginal effect of pollution on the growth rate of human capital is larger than that on the growth rate of physical capital. Pollution impacts human capital in two ways. First, pollution shifts expenditures away from private education because pollution reduces longevity through $\phi(z_t)$, which in turn decreases the agent's effective altruism. Second, pollution undermines the effectiveness of education expenditures through $\lambda(z_t)$. In contrast, pollution only has its impact on physical capital through agents' propensity to save, which depends on the agent's longevity $\phi(z_t)$.

Example 2, $\varepsilon = -2.5$. The longevity function is $\phi(z_t) = \frac{1}{1+z_t^3}$ and the education expenditures effectiveness function is $\lambda(z_t) = \frac{1}{1+z_t^{0.5}}$.

[Insert Figure 5 here]

In Figure 5, the upper panel illustrates the shapes of longevity $\phi(z_t)$ and education expenditures effectiveness $\lambda(z_t)$. The middle panel shows the phase diagram and local dynamics of the unique BGP. When the stock of pollution is below the threshold, the *kk* locus first rises for lower stock of pollution but then falls for higher stock of pollution. This is because for a lower stock of pollution, $\lambda(z_t)$ is steeper than $\phi(z_t)$. As the stock of pollution increases, however, $\phi(z_t)$ becomes steeper than $\lambda(z_t)$. So the negative marginal effect of pollution on the growth rate of human capital is initially larger and then smaller than that on the growth rate of physical capital. The *zz* locus intersects with the *kk* locus when the *kk* locus slopes down.

It is interesting in this example that economic and environmental cycles emerge that echo with those cycles highlighted in Figure 2. The BGP features locally dampened cycles (the associated eigenvalues are $0.167 \pm 0.169i$). The emergence of cycles lies in the asymmetrical health effects of pollution on physical and human capital. To understand the intuition behind the cycles, suppose the health effects of pollution are absent for a moment. From the difference equation for physical capital (18a) and the difference equation for human capital (19a), if the ratio of physical to human capital k_t is larger than the value on the BGP, the accumulation of physical capital is slower than that of human capital, such that the value for k_t monotonically decreases and converges towards the value on the BGP. When the asymmetrical health effects of pollution come into force, however, the convergence of a higher k_t towards the BGP may not necessarily be monotonic. Rather, the economic and environmental systems cyclically converge towards the BGP. The key to understanding the cycles lies in the dynamic interactions between k_t and z_t. The stock of pollution, z_t, increases in the ratio of physical to human capital, k_t, because physical capital is "dirty" and human capital is "clean". In contrast, the ratio of physical to human capital,

k_t, decreases in the stock of pollution because pollution damages the accumulation of physical capital more than that of human capital. A higher value for k_t generates more pollution, which in turn causes the accumulation of physical capital to become temporarily slower than that of human capital. Thus, the value for k_t decreases and less pollution is generated. Physical capital temporarily accumulates faster than human capital does, and thus k_t increases. The cycle repeats itself as the economic and environmental systems converge towards the BGP.

The economic and environmental cycles represent aggregate volatility and inequality between generations, and government policy is required to eliminate the cycles (Seegmuller and Verchère, 2004; Schumacher and Zou, 2015; Palivos and Varvarigos, 2017; Raffin and Seegmuller, 2017). If the government raises the tax rate from $\tau = 0.05$ to $\tau = 0.06$, as shown in the lower panel of Figure 5, the zz locus intersects with the kk locus when the kk locus slopes up, the BGP is locally stable without cycles (the associated eigenvalues become 0.25 and 0.08). So by raising the tax rate on the output, the government is able to reduce the volatility and intergenerational inequality associated with the transitional dynamics to the BGP. The reason why the government policy works to eliminate cycles also rests on the health effects of pollution and on the dynamic interactions between k_t and z_t. As the government shifts the BGP to the area where pollution undermines human capital more than physical capital, k_t changes with z_t in the same direction. In addition, z_t also changes with k_t in the same direction because k_t measures the abundance of dirty capital relative to clean capital. So either the combination of high z_t and high k_t or the combination of low z_t and low k_t can be sustained on the BGP, and no cycles emerge.

Example 3, $\varepsilon = 2.5$. The longevity function is $\phi(z_t) = \frac{1}{1+z_t^3}$ and the education expenditures effectiveness function is $\lambda(z_t) = \frac{1}{1+z_t^{5.5}}$.

[Insert Figure 6 here]

This example features another interesting case with multiple BGPs that conforms to the negative relationship between pollution and economic growth revealed in Figure 1. In the upper panel of Figure 6, as the stock of pollution increases, the slope of $\lambda(z_t)$ becomes increasingly steeper relative to that of $\phi(z_t)$. As the stock of pollution increases further, $\phi(z_t)$ becomes relatively steeper because $\lambda(z_t)$ converges to its lower bound but $\phi(z_t)$ still decreases in pollution. In the middle panel of Figure 6, below the threshold stock of pollution, the *kk* locus is convexly increasing and intersects with the *zz* locus twice at *A* and *B*. BGP *A* is locally stable with eigenvalues 0.40 and -0.07. BGP *B* exhibits locally saddle stability with eigenvalues 1.28 and -0.95. Above the threshold stock of pollution, the slope of the *kk* locus becomes smaller because the negative effect of pollution on human capital cannot become any larger but the negative effect on physical capital still grows. So the *kk* locus intersects with the *zz* locus again at *C*. BGP *C* is also locally stable with eigenvalues 0.72 and -0.39.

Due to the local dynamic properties, BGP *B* gives rise to a separatrix. This separatrix along with the threshold stock of pollution serve as a boundary that demarcates the first quadrant into two "sink" regions. The points to the left of the boundary will converge to BGP *A*, whereas the points to the right of the boundary will converge to BGP *C*. Policymakers strictly prefer BGP *A* to BGP *C*. BGP *A* features a high economic growth rate and a low stock of pollution, and is thus desirable. In contrast, BGP *C* features a low economic growth rate and a high stock of pollution. So BGP *C* should be avoided. We summarize the ranking of multiple BGPs in the following proposition.

Proposition 6. *When multiple BGPs emerge, the BGP under the PE regime is preferred over the BGP under the NPE regime because the economic growth rate is higher and the stock of pollution is lower on the BGP under the PE regime than under the NPE regime.*

Proof. See Appendix F. ☐

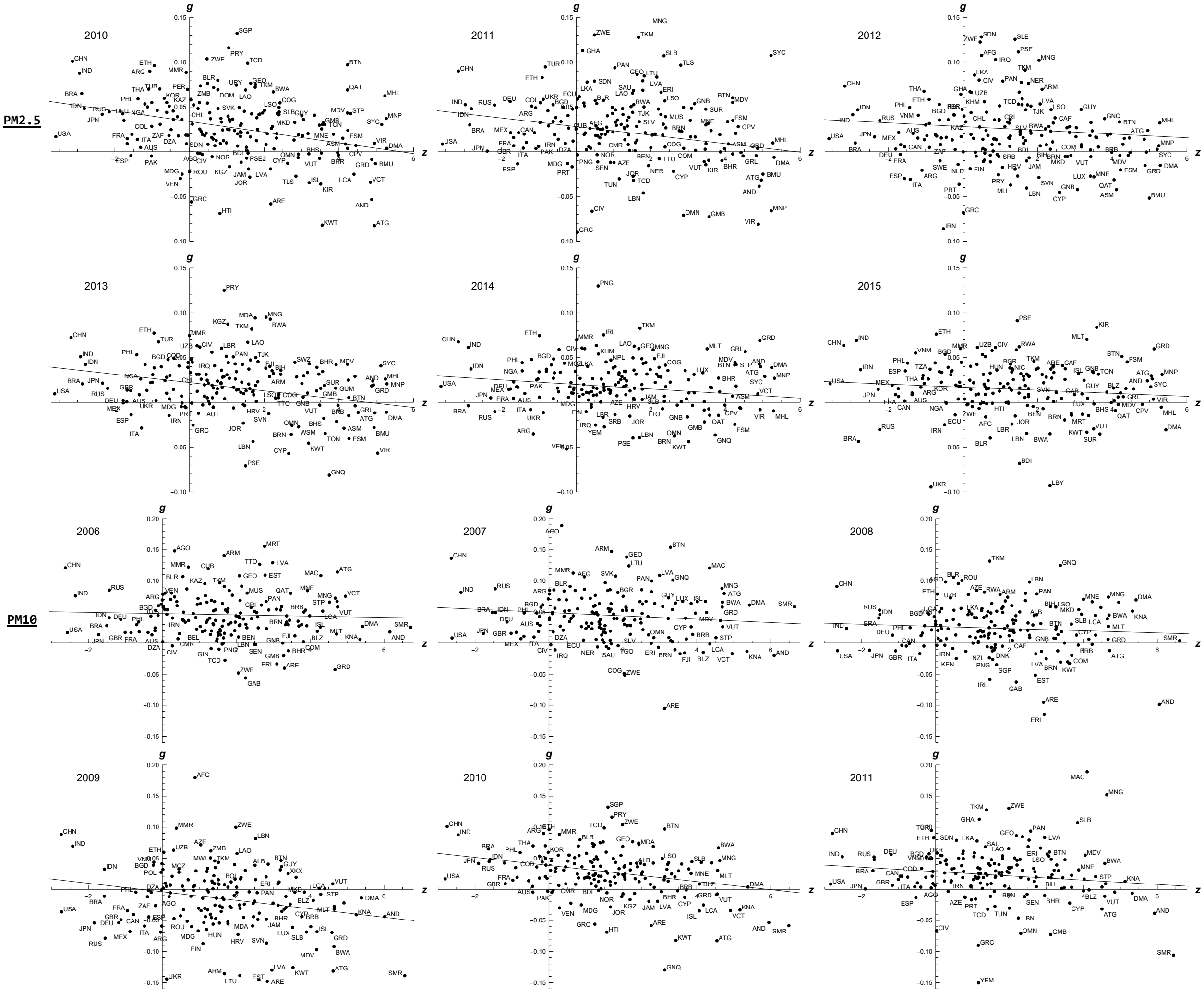
In terms of policy, if the economy lies to the right of the boundary, policymakers need to steer the economy to the left of the boundary such that the economy will converge to the desirable BGP in the long run. In the lower panel of Figure 6, the dashed lines represent the loci when $\tau = 0.05$, and the solid lines represent the loci when $\tau = 0.052$ (a 4% increase in the tax rate). As the BGP in the middle shifts from *B* to B', a new separatrix is generated. Point *D* initially lies to the right of the old separatrix and to the left of the threshold stock of pollution. If there were no government intervention, the local dynamics are dictated by (20a) and (16), and z_t will increase until it jumps over the threshold stock of pollution. As the local dynamics are dictated by (20b) and (16) instead, the economy converges to the undesirable BGP C in the long run. If the government raises the tax rate, however, point D lies to the left of the new separatrix and the economy will converge to the desirable BGP A in the long run. As an alternative measure, the government can decrease the ratio of physical to human capital by encouraging agents to increase private education expenditures, decrease savings, or both. Point D will jump over the old separatrix to reach point E. The economy will converge to the desirable BGP A in the long run without the government modifying the tax rate τ.

6 Conclusion

Motivated by the empirical observations that pollution and economic growth can be negatively correlated and that environmental and economic cycles can emerge, we argue that a possible explanation lies in how pollution and the ratio of physical to human capital affect each other. We thus develop an otherwise standard overlapping generations model that considers the dynamic interactions between pollution and the economy. Pollution affects the economy due to its asymmetrical health effects on physical and human capital. If pollution more negatively affects the accumulation of human capital than that of physical capital, the ratio of physical to human capital increases in pollution. But if pollution more negatively affects the accumulation of physical capital than that of human capital, the ratio of physical to human capital decreases in pollution. The economy affects pollution due to the nature of the two types of capital. As physical capital generates pollution while human capital does not, the stock of pollution increases in the ratio of physical to human capital. Our model specification contributes to the theoretical literature by filling the gap between the health effect of pollution on the accumulation of physical capital and that on the accumulation of human capital.

The patterns of economic dynamics arising from our model are in line with real-world empirical observations. One interesting case may emerge with two extreme BGPs separated by a separatrix and a threshold stock of pollution. One BGP is strictly preferred over the other because the superior one features a lower stock of pollution and a higher economic growth rate. This result explains the negative trend lines in the cross-sectional data sets shown in Figure 1. We have shown that government intervention matters for the economy to converge to the desirable BGP in the long run. In another interesting case, the ratio of physical to human capital first increases and then decreases in the stock of pollution, and the economy may experience dampened cycles. This result replicates the cycles outlined in the time-series data sets shown in Figure 2. We have shown that the government can avoid the cycles and reduce volatility by levying a higher tax rate.

Our theoretical results highlight the importance of understanding how the accumulation of physical capital is negatively affected by pollution relative to that of human capital. We note that there is a lack of empirical evidence documenting the relative health effects and call for future research that estimates the relative health effects of specific pollutants.

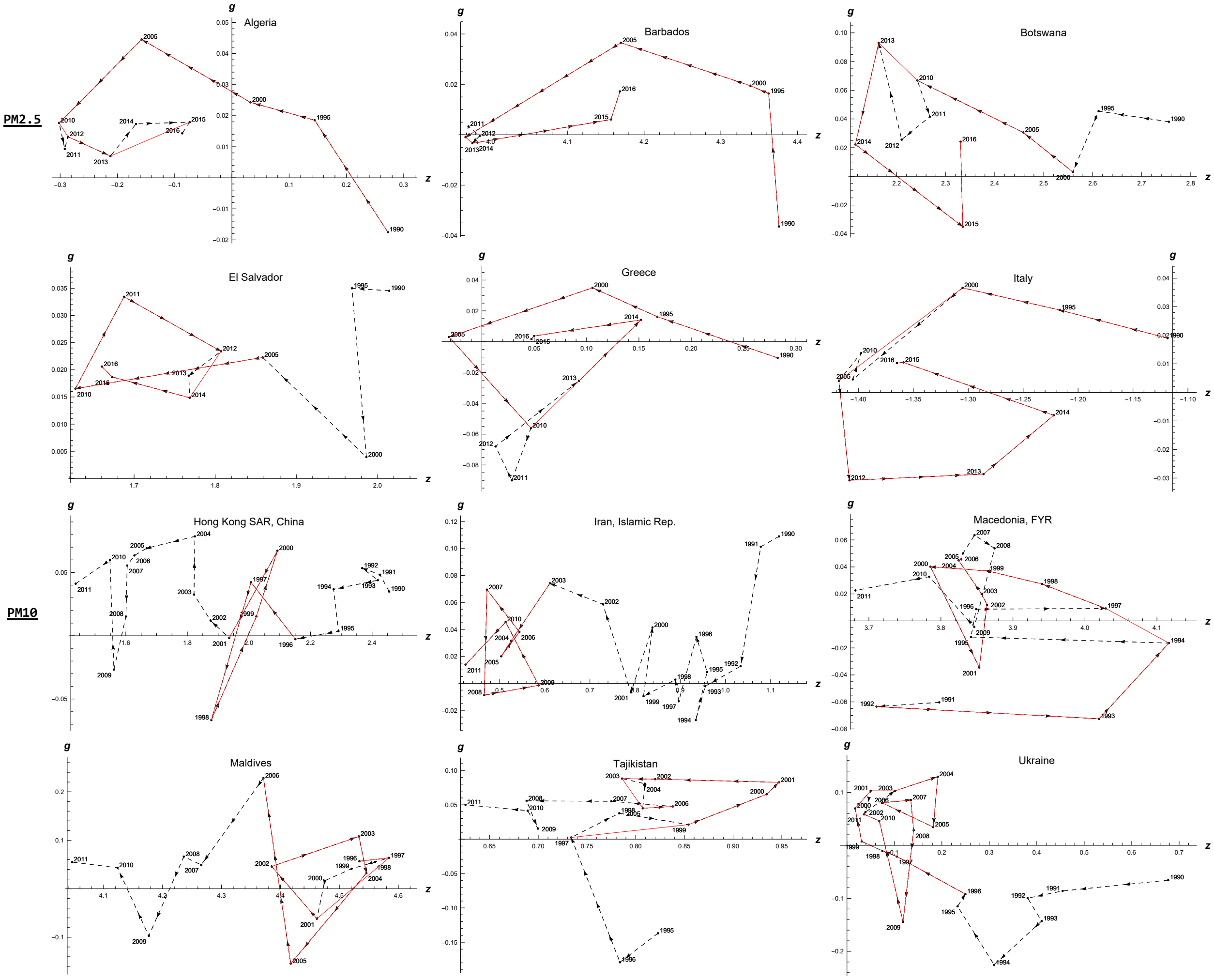


Notes. (1) In each figure cell, pollution (z) on the horizontal axis is represented by air stock pollutants, which are the logged values of population-weighted PM2.5 (the first two rows, 2010-2015) and PM10 (the last two rows, 2006-2011). Economic growth (g) on the vertical axis is the annual growth rate of real GDP per capita.

(2) The country codes can be found in World Bank Group (2018a).

(3) The trend lines are drawn based on least-squares fits to scatters of countries and regions in each year.

Figure 1: The Negative Relationship between Pollution and Economic Growth



Notes. (1) In each figure cell, pollution (z) on the horizontal axis is represented by air stock pollutants, which are the logged values of population-weighted PM2.5 (the first two rows) and PM10 (the last two rows). Economic growth (g) on the vertical axis is the annual growth rate of real GDP per capita.

(2) The cycles are colored red after some points are smoothed if necessary. For example, to highlight the cycle of PM2.5 and economic growth in Algeria (the upper-left corner), we smooth the cycle by omitting the points for 2011 and 2014.

Figure 2: The Cycles of Pollution and Economic Growth for Selected Countries

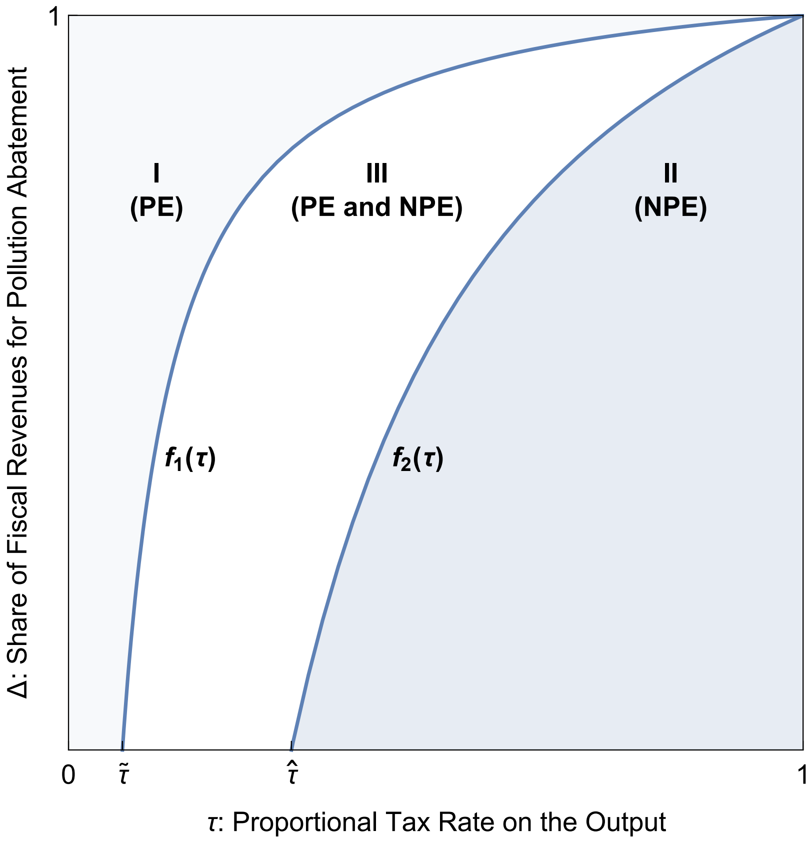
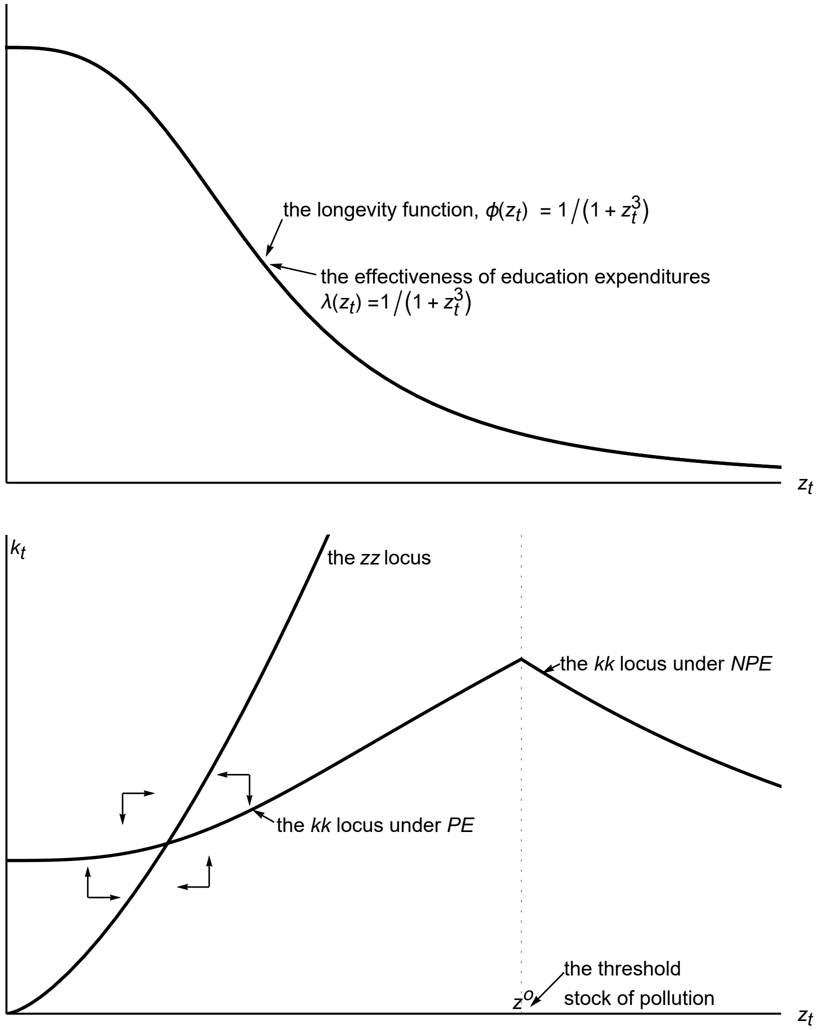


Figure 3: Three Regions in the Policy Set



Note. In this and the following phase diagrams, we plot the stock of pollution, z_t, on the horizontal axis, and the ratio of physical to human capital, k_t, on the vertical axis.

Figure 4: The Benchmark Phase Diagram, $\varepsilon = 0$

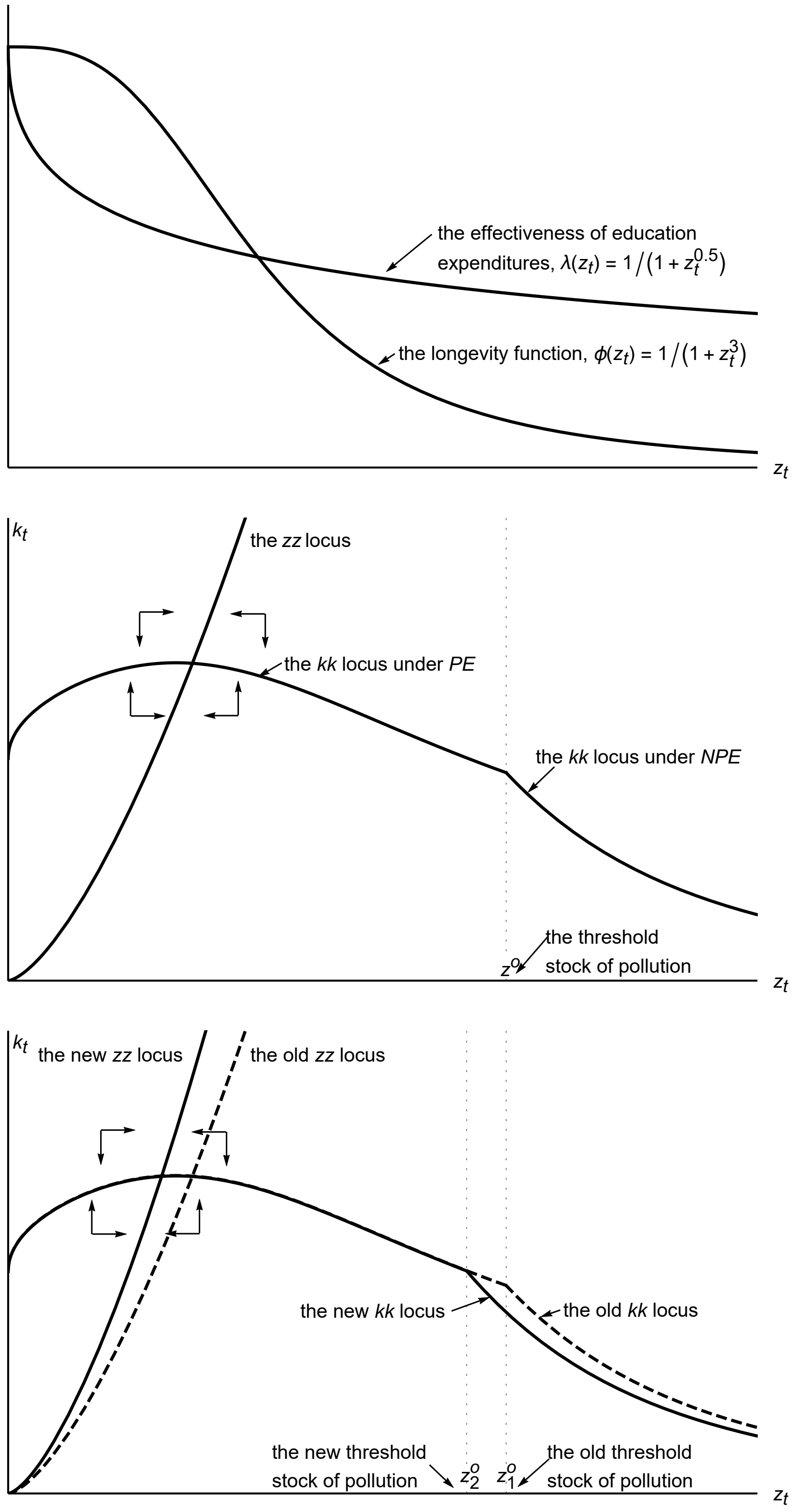


Figure 5: The Phase Diagram and the Effects of Government Policy, $\varepsilon = -2.5$

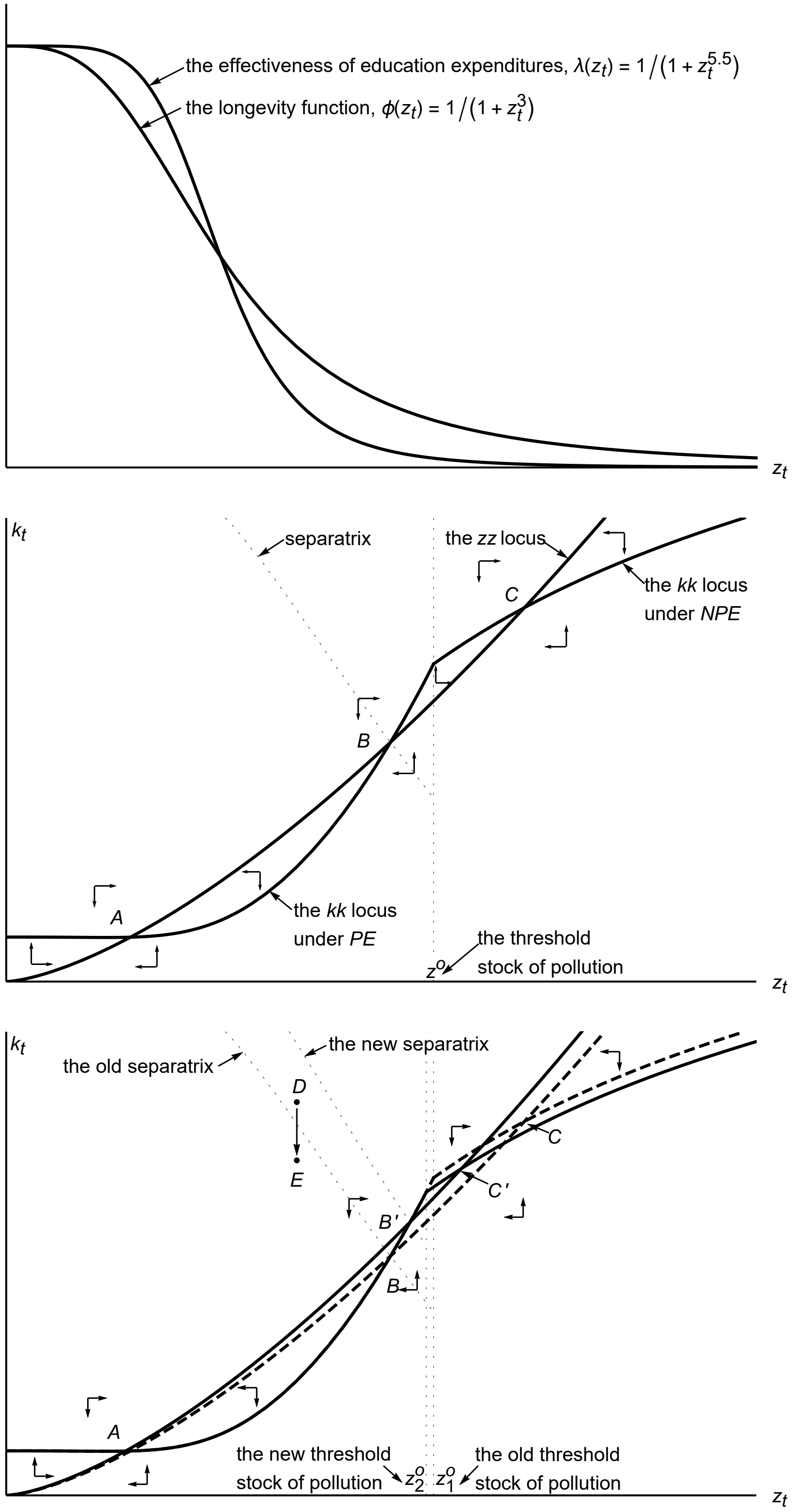


Figure 6: The Phase Diagram and the Effects of Government Policies, $\varepsilon = 2.5$

Table 1: The Benchmark Parameters

Category	Description	Parameter	Value
Production	Production function scalar	A	10
	Physical capital's share in production	α	0.33
Environment	The dissipation rate of the stock of pollution	θ	0.8
	The polluting capacity of physical capital	ρ	0.2
Longevity*	Lower bound of longevity	$\underline{\phi}$	0
	Upper bound of longevity	$\overline{\phi}$	1
	Curvature of the longevity function	c_ϕ	3***
Education spending effectiveness**	Lower bound of the effectiveness	$\underline{\lambda}$	0
	Upper bound of the effectiveness	$\overline{\lambda}$	1
	Curvature of education spending effectiveness	c_λ	3***
Human capital	Scalar in the evolution of human capital	B	5
	Education expenditure's share in human capital	β	0.6
	The relative strength of public to private education	μ	1
Utility	Agents' altruism	χ	0.65
Government	Proportional tax on final output	τ	0.05
	Pollution abatement's share in fiscal revenues	Δ	0.5

Notes. *The longevity function is $\phi(z_t) = \frac{\overline{\phi}+\underline{\phi} z_t^{c_\phi}}{1+z_t^{c_\phi}}$.

**The education expenditures effectiveness function is $\lambda(z_t) = \frac{\overline{\lambda}+\underline{\lambda} z_t^{c_\lambda}}{1+z_t^{c_\lambda}}$.

***So the differential between the two curvature parameters, ε, is equal to 0 in the benchmark because $\varepsilon = c_\lambda - c_\phi = 3 - 3 = 0$.

References

Agénor, P.-R. (2011). Schooling and public capital in a model of endogenous growth. *Economica*, 78(309):108–132.

Agénor, P.-R. and Neanidis, K. C. (2011). The allocation of public expenditure and economic growth. *The Manchester School*, 79(4):899–931.

Aloi, M. and Tournemaine, F. (2013). Inequality, growth, and environmental quality trade-offs in a model with human capital accumulation. *Canadian Journal of Economics/Revue canadienne d'économique*, 46(3):1123–1155.

Barro, R. J. (1990). Government spending in a simple model of endogeneous growth. *Journal of Political Economy*, 98(5):S103–S125.

Barro, R. J. (2001). Human capital and growth. *The American Economic Review*, 91(2):12–17.

Bloom, D. E., Canning, D., and Graham, B. (2003). Longevity and life-cycle savings. *Scandinavian Journal of Economics*, 105(3):319–338.

Buiter, W. H. and Kletzer, K. M. (1995). Capital mobility, fiscal policy, and growth under self-financing of human capital formation. *The Canadian Journal of Economics / Revue canadienne d'Economique*, 28:S163–S194.

Chen, S., Guo, C., and Huang, X. (2018a). Air pollution, student health, and school absences: Evidence from China. *Journal of Environmental Economics and Management*, 92:465–497.

Chen, S., Oliva, P., and Zhang, P. (2018b). Air pollution and mental health: Evidence from China. *Retrieved from SSRN: https://ssrn.com/abstract=3178649 or http://dx.doi.org/10.2139/ssrn.3178649*.

Currie, J., Hanushek, E. A., Kahn, E. M., Neidell, M., and Rivkin, S. G. (2009). Does pollution increase school absences? *Review of Economics and Statistics*, 91(4):682–694.

De Nardi, M., French, E., and Jones, J. B. (2009). Life expectancy and old age savings. *The American Economic Review*, 99(2):110–115.

Devarajan, S., Swaroop, V., and Zou, H.-f. (1996). The composition of public expenditure and economic growth. *Journal of Monetary Economics*, 37(2):313–344.

Duczynski, P. (2002). Adjustment costs in a two-capital growth model. *Journal of Economic Dynamics and Control*, 26(5):837–850.

Duczynski, P. (2003). On the empirics of the imbalance effect. *International Journal of Business and Economics*, 2(2):121–128.

Ebenstein, A., Fan, M., Greenstone, M., He, G., Yin, P., and Zhou, M. (2015). Growth, pollution, and life expectancy: China from 1991-2012. *American Economic Review*, 105(5):226–231.

Ebenstein, A., Lavy, V., and Roth, S. (2016). The long-run economic consequences of high-stakes examinations: Evidence from transitory variation in pollution. *American Economic Journal: Applied Economics*, 8(4):36–65.

Economides, G. and Philippopoulos, A. (2008). Growth enhancing policy is the means to sustain the environment. *Review of Economic Dynamics*, 11(1):207–219.

Fodha, M. and Seegmuller, T. (2014). Environmental quality, public debt and economic development. *Environmental and Resource Economics*, 57(4):487–504.

Gradus, R. and Smulders, S. (1993). The trade-off between environmental care and long-term growth—Pollution in three prototype growth models. *Journal of Economics*, 58(1):25–51.

Jouvet, P.-A., Pestieau, P., and Ponthiere, G. (2010). Longevity and environmental quality in an OLG model. *Journal of Economics*, 100(3):191–216.

Kim, Y., Manley, J., and Radoias, V. (2017). Air pollution and long-term mental health. *Retrieved from SSRN: https://ssrn.com/abstract=3028930 or http://dx.doi.org/10.2139/ssrn.3028930.*

Ladrón-de Guevara, A., Ortigueira, S., and Santos, M. S. (1997). Equilibrium dynamics in two-sector models of endogenous growth. *Journal of Economic Dynamics and Control*, 21(1):115–143.

Maher, B. A., Ahmed, I. A. M., Karloukovski, V., MacLaren, D. A., Foulds, P. G., Allsop, D., Mann, D. M. A., Torres-Jardón, R., and Calderon-Garciduenas, L. (2016). Magnetite pollution nanoparticles in the human brain. *Proceedings of the National Academy of Sciences*, 113(39):10797–10801.

Motoyama, T. (2016). From physical to human capital accumulation with pollution. *Retrieved from http://www2.econ.osaka-u.ac.jp/library/global/dp/1603.pdf.*

Mulligan, C. B. and Sala-i Martin, X. (1993). Transitional dynamics in two-sector models of endogenous growth. *The Quarterly Journal of Economics*, 108(3):739–773.

Nilsson, J. (2009). The long-term effects of early childhood lead exposure: Evidence from the phase-out of leaded gasoline. *Retrieved from http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.212.3578.*

Osang, T. and Sarkar, J. (2008). Endogenous mortality, human capital and economic growth. *Journal of Macroeconomics*, 30(4):1423–1445.

Palivos, T. and Varvarigos, D. (2017). Pollution abatement as a source of stabilzation and long-run growth. *Macroeconomic Dynamics*, 21(3):644–676.

Pautrel, X. (2009). Pollution and life expectancy: How environmental policy can promote growth. *Ecological Economics*, 68(4):1040–1051.

Pautrel, X. (2012). Pollution, private investment in healthcare, and environmental policy. *The Scandinavian Journal of Economics*, 114(2):334–357.

Raffin, N. (2012). Children's environmental health, education, and economic development. *Canadian Journal of Economics/Revue canadienne d'économique*, 45(3):996–1022.

Raffin, N. and Seegmuller, T. (2014). Longevity, pollution and growth. *Mathematical Social Sciences*, 69:22–33.

Raffin, N. and Seegmuller, T. (2017). The cost of pollution on longevity, welfare and economic stability. *Environmental and Resource Economics*, 68(3):683–704.

Sapci, O. and Shogren, J. F. (2017). Environmental quality, human capital and growth. *Journal of Environmental Economics and Policy*, 7(2):184–203.

Schumacher, I. and Zou, B. (2015). Threshold preferences and the environment. *Journal of*

Mathematical Economics, 60:17–27.

Seegmuller, T. and Verchère, A. (2004). Pollution as a source of endogenous fluctuations and periodic welfare inequality in OLG economies. *Economics Letters*, 84(3):363–369.

Smulders, S. and Gradus, R. (1996). Pollution abatement and long-term growth. *European Journal of Political Economy*, 12(3):505–532.

Varvarigos, D. (2010). Environmental degradation, longevity, and the dynamics of economic development. *Environmental and Resource Economics*, 46(1):59–73.

Varvarigos, D. (2013a). Endogenous longevity and the joint dynamics of pollution and capital accumulation. *Environment and Development Economics*, 19(04):393–416.

Varvarigos, D. (2013b). Environmental dynamics and the links between growth, volatility and mortality. *Bulletin of Economic Research*, 65(4):314–331.

Wen, M. and Gu, D. (2012). Air pollution shortens life expectancy and health expectancy for older adults: The case of China. *Journals of Gerontology - Series A Biological Sciences and Medical Sciences*, 67(11):1219–1229.

World Bank Group (2018a). The world development indicators. *Retrieved from https://datacatalog.worldbank.org/dataset/world-development-indicators*.

World Bank Group (2018b). The world development indicators database archives. *Retrieved from https://databank.worldbank.org/data/reports.aspx?source=1277&series=EN.ATM.PM10.MC.M3*.

World Health Organization (2018). Global ambient air quality database. *Retrieved from https://www.who.int/airpollution/data/cities/en/*.

Zhang, J. and Zhang, J. (2005). The effect of life expectancy on fertility, saving, schooling and economic growth: Theory and evidence. *Scandinavian Journal of Economics*, 107(1):45–66.

Zhang, X., Zhang, X., and Chen, X. (2017). Happiness in the air: How does a dirty sky affect mental health and subjective well-being? *Journal of Environmental Economics and Management*, 85:81–94.